International Trade and Supply Function Competition*

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Abstract

In recent Ricardian and heterogeneous firm models of international trade, probabilistic representations of technologies have been very popular. Using a similar setup, we propose a model in which exporters compete in supply functions/schedules. Our model reconciles the existence of multiple sellers, multiple prices, and variable markups while also incorporating novel features such as strategic pricing and incomplete information. Moreover, the probabilistic structure in trade models usually comes with strong parametric assumptions on the productivity distributions (usually assumed to be Fréchet or Pareto) of countries/firms. Recent studies have shown that gains from trade predictions are very sensitive to these parametrizations. In view of this, our model maintains a flexible structure for productivity distributions and marginal costs. We recover the productivity distributions and the marginal cost functions *nonparametrically*. The model is estimated using disaggregated bilateral trade data, which only consists of trade values and traded quantities. Our empirical results do not support the usual distributional assumptions. In particular, we find that the productivity distributions are not unimodal. In addition, our results show that markups are not constant across exporters thereby adding to the recent literature with variable markups. Our methodology also contributes to the empirical share auction literature by showing that the underlying structure is identified nonparametrically even if we do not observe the entire schedules, but only the *transaction point* instead.

Keywords: trade, technology, supply function competition, nonparametric identification and estimation, divisible good or share auctions

1 Introduction

In most of the recent papers in international trade literature using Ricardian or heterogeneous firm models with a probabilistic representation of technologies, which allows for technological differences across countries/firms, either perfect competition, Bertrand competition or monopolistic competition is assumed. Looking at the bilateral trade data that have been used in some of these models, which consists of trade values and traded quantities, even in the least aggregated level we observe the following: First, within the same product category we see many exporters for a given destination. Each supplies a share of that market, thus one exporter does not swipe the whole market. Baldwin and Harrigan (2010) reports that for US imports even in the 10-digit Harmonized System Codes, which is the least aggregated data publicly available, the median number of countries is 12. Second, again within the same product category and for a given destination, for all the exporters we see different unit values, which are the ratio of the trade value to traded quantity. This suggests the existence of different prices.

In Eaton and Kortum (2002), the underlying market structure is perfect competition. This implies the lowest cost exporter should supply the whole import market which is not consistent with the first feature of the data. Since perfect competition also implies marginal cost pricing, Bernard, Eaton, Jensen and Kortum (2003) assume Bertrand competition in an attempt to improve on this to allow for variable markups. This still suggests, however, that the lowest cost exporter supplies the whole market. One possible explanation for both of the features of trade data mentioned above is a market structure where there is monopolistic competition, where each firm produces a differentiated product.[1] In Melitz (2003), Helpman, Melitz and Yeaple (2004), Chaney (2008), Helpman, Melitz and Rubinstein (2008) and many other heterogeneous firm models, we see this type of structure which has the following implications: First, monopolistic competition with Dixit-Stiglitz preferences implies standard

[1] For the analysis of countries as exporters, one can interpret this as countries with technological differences producing differentiated goods.

mark-up pricing where markups are constant across exporters and markets. This implication has been questioned and there have been studies which attempt to account for variable markups such as Melitz and Ottaviano (2008) in which variable markups are obtained from the demand side by imposing a more specific structure on preferences or Bernard, Eaton, Jensen and Kortum (2003) in which they are obtained from the supply side à la Bertrand competition. Second, the usual implication of these monopolistic competition models is that the pricing decision of one exporter does not influence the pricing decision of the others.

In this paper we propose an alternative way of modelling the exporting behavior of countries/firms which can explain the aforementioned features of data while still allowing for variable markups and the fact that the pricing decision of a country/firm can actually influence the decision of the others. Inspired by both Eaton and Kortum (2002) and Klemperer and Meyer (1989) we propose a model in which exporters compete in supply functions/schedules. According to Klemperer and Meyer (1989) in the presence of any uncertainty, firms might not want to commit to a fixed price or a fixed quantity.[2] Some decisions, however, such as size and structure, have to be made well in advance before this uncertainty is resolved. Such decisions, as indicated by Klemperer and Meyer (1989) , would implicitly determine a supply function relating the quantity that will be sold at any given price. They further argue that by offering supply functions, firms can make higher expected profits since it provides a better adaptation to uncertainty. In the context of international trade, it is even more likely that the exporters will face a lot of uncertainty since trade involve relations beyond the national borders. In supply function competition, each country/firm offers a supply schedule, which are horizontally added to obtain the aggreagte supply. Market clearing price is determined where this aggregate supply meets aggregate demand. The intersection of the market clearing price with the individual supply function gives how many units will be supplied by that country/firm. If there is uniform pricing, the revenue is this quantity multiplied by market clearing price whereas if there is discriminatory pricing the revenue is the area under the

[2]Our model differs from Atkeson and Burstein (2008) in this aspect since their model, in which they have variable markups and quantity competition à la Cournot, implies committing to a fixed quantity.

supply schedule upto this quantity. Since we observe multiple unit values across exporters for a given product in a given destination, we assume discriminatory pricing.

Following the probabilistic representation of technologies in Eaton and Kortum (2002), countries/firms draw their productivity from some productivity distribution. In our case, however, this information will be private so that countries can only observe their own productivity but not the others'. Even with the advanced information technologies of today, monitoring the behavior of all the rival countries in the production of every single industry would be extremely costly and still might not even be possible. In this aspect our model differs from Klemperer and Meyer (1989) in which there is complete information about costs. We follow Wilson (1979) to introduce incomplete information. Hortacsu and Puller (2008) and Vives (2008) also follow Wilson (1979) to study supply function competition with private information under uniform pricing.

The structure in our model (other than supply function competition environment) is similar to Eaton and Kortum (2002). Countries are assumed to draw their productivities for each good from a source-destination specific productivity distribution.[3] Similar to their model, marginal cost is also source-destination specific. In almost all of the Ricardian and heterogeneous firm models, strong parametric assumptions are made on these productivity distributions and marginal costs which provide analytical solutions and tractable equations, making these models empirically relevant in explaining various factors affecting trade flows.[4] In contrast to the existing literature, we will not make any parametric assumptions on the productivity distribution and marginal cost function.

Why is this important ? This will allow us to check the adequacy of the parametric assumptions in the existing trade literature, which naturally affects the predictions of those models such as gains from trade and related policy questions. For instance, Arkolakis, Costinot and Rodriguez-Clare (2010) argue that one parameter of the productivity distrib-

[3] In Eaton and Kortum (2002) this distribution is only source country specific.

[4] Because there is no empirical application in Melitz (2003), his theoretical results are derived for a general productivity distribution.

ution is actually one of the only two statistics governing gains from trade in most of these models. In a recent study, Simonovska and Waugh (2010) show that incorrect estimates of that parameter causes the gains from trade to be estimated half of what it should be. If we can recover the underlying structure (productivity distribution and marginal cost) from data with minimal parametric assumptions this might shed light on such concerns.

Recent literature of empirical share auctions provides insights about how to recover the productivity distributions and marginal costs nonparametrically. Note that our environment is very similar to the divisible good auction (share auction) environment in which instead of sellers, buyers compete. In a share auction there is a divisible item (or identical units of the same item) and each bidder submits a bid schedule (demand schedule) which shows at every price how many units (what share) she wants to have. Just as there is an underlying marginal cost determining the supply schedule in our model, there is an underlying marginal valuation determining this demand schedule. Some part or all of this marginal valuation can be private information just as productivities as part of marginal cost are private information in our model. Hortacsu (2002) studies discriminatory share auction of treasury bills, whereas Hortacsu and Kastl (2008), Kastl (2010) consider uniform pricing and recover the unknown marginal valuations following the approach in Guerre, Perrigne and Vuong (2000).

In this paper, we show that we can identify the productivity distributions and the marginal cost nonparametrically. Our methodology contributes to the literature of empirical share auctions in two important aspects: First, in Hortacsu (2002), Kastl (2010) and others, they usually have data on the bid schedules of the bidders whereas this is not the case in our data. In our data we can only observe the transaction/equilibrium points. Nevertheless, we show that the underlying structure, namely productivity distribution and cost function is identified, when one observes only the transaction/equilibrium points. Second, those papers deal with the asymmetry among the bidders by creating groups in which bidders in the same group are considered to be symmetric. In this paper, we allow for asymmetry without defining groups. In empirical applications of supply function competition model as in Wolak

(2003a, 2003b) and Hortacsu and Puller (2008) where they study competition in electricity markets they also observe the supply schedule of the firms. Availability of the individual demand/supply schedule, however, is a very specific characteristic of these type of markets. The fact that we can identify the underlying structure by observing only the transaction points makes the supply function competition model applicable to other markets as well.

In Eaton and Kortum (2002) and Bernard, Eaton, Jensen and Kortum (2003), the productivity distributions are assumed to be Fréchet. In others such as Helpman, Melitz and Yeaple (2004), Melitz and Ottaviano (2008), Helpman, Melitz and Rubinstein (2008), Eaton, Kortum and Kramarz (2010) and many others the productivity distribution is assumed to be Pareto. Fréchet distribution is also called the Type II extreme value distribution since it is related to the asymptotic distribution of the largest value. Eaton and Kortum (2002) refers to Kortum (1997) and Eaton and Kortum (1999) where they show how certain processes of innovation and diffusion give rise to this type of distribution. They argue that while producing any good, the actual technique that would ever be used in a country represents the best discovered one to date so it is reasonable to represent technology with an extreme value distribution. The models in Eaton and Kortum (2002) and Bernard, Eaton, Jensen and Kortum (2003), are mainly concerned with the best producers of a country for each good, which is considered as the explanation of the choice of an extreme value type for a country's productivity distribution. On the other hand, in Helpman, Melitz and Yeaple (2004), Melitz and Ottaviano (2008), Helpman, Melitz and Rubinstein (2008), Eaton, Kortum and Kramarz (2010) the main concern is not necessarily the best producers, which justifies the choice of an "non-extreme" value type of distribution such as Pareto. Helpman, Melitz and Yeaple (2004) refer to Axtell (2001) for justification, where he shows that US firm size distribution closely follows a Pareto distribution. This, however, does not say much about the underlying productivity distribution.[5] Both distributional assumptions provide great analytical

[5] It is worth mentioning the link between Pareto and Fréchet: The limiting distribution of the maximum of independent random variables having Pareto distribution is Frechet which suggests that when all firms draw from Pareto the distribution of the best can be represented as Fréchet.

convenience in these models.

We apply our model to German market for manufacturing imports for 1990. We use the same year as Eaton and Kortum (2002). Our empirical results do not support those distributional assumptions. Another important observation is that the productivity distributions are not unimodal. Low productivities are more likely to occur as expected, but there is not a single mode. This is important to consider from a policy perspective since it suggests certain industries have certain types (levels) of productivity. Policy makers might want to distinguish between less productive and more productive types. In addition, in contrast to the monopolistic competition models with Dixit Stiglitz preferences, which implies constant markups across exporters, our empirical results suggest that they are not constant.

The paper is organized as follows: Section 2 introduces our model. Section 3 establishes the nonparametric identification of productivity distributions and the marginal cost functions. Section 4 introduces data and explains how to control for differences in measurement units and heterogeneity across goods. Also, in Section 4 we explain how to construct the market clearing prices. Section 5 describes our estimation procedure. Readers who are interested in the empirical results may skip Section 3 and Section 5 and move directly to Section 6, which provides our empirical results. The appendix contains the proof of some results.

2 The Model

Consider a world with N countries, indexed by $n = 1, 2, ..., N$ and a finite number, J, of goods indexed by $j = 1, 2, ..., J$. In each country n for any good j, the other $N-1$ countries, indexed by $i = 1, 2, .., n-1, n+1, .., N$, are competing with each other to export good j to country n. The index n will refer to destination/importer country or market whereas index i will refer to source/exporter country or seller.

Following the probabilistic representation of technologies in Eaton and Kortum (2002), country i's productivity, z_{ni}^{j}, in producing good j for destination n, is the realization of a random variable Z_{ni}^{j} independently drawn across j from a destination-source specific distribution, $F_{ni}(\cdot)$ which is absolutely continuous with density $f_{ni}(\cdot)$ and support $[\underline{z}_{ni}, \overline{z}_{ni}] \subseteq \mathbb{R}_{+}$.[6] In Eaton and Kortum (2002), this distribution is only source country specific. Our model we allows for the more general case where this distribution is destination-source specific. This allows us to test whether a country's productivity distribution differs across destinations. For instance, if it differs checking the exporter composition of those destinations might provide valuable insights. Also, following the convention in the literature, these productivities are independent across countries. Thus, we make the following assumption.

Assumption A1 :

(i) For any given n and i, Z_{ni}^{j}'s are drawn independently from $F_{ni}(\cdot)$ across j.

(ii) For all n and for all $i, i', i \neq i'$, $\left\{Z_{ni}^{j}\right\}_{j=1}^{J}$ is independent of $\left\{Z_{ni'}^{j}\right\}_{j=1}^{J}$.

In contrast to the conventional approach in the literature, we are not making any parametric assumptions on these productivity distributions.

For any country i, the marginal cost of producing the qth unit of good j for destination n is denoted by $c_{ni}^{j}(q, b^{j}, z_{ni}^{j})$ where b^{j} is a vector of characteristics of good j such as being a high tech good, luxury good, unit of measure of the good, world price of the good, etc.

Assumption A2 : *For all n, i and j, marginal cost*

$$c_{ni}^{j}(q, b^{j}, z_{ni}^{j}) = \frac{c_{ni}^{\circ}(q, b^{j})}{z_{ni}^{j}}$$

where $c_{ni}^{\circ}(\cdot,\cdot)$ is continuously differentiable and weakly increasing in q, for all $b^{j} \in \mathbb{R}^{m}$, $q \geq 0$.

Given good characteristics, the functional form $c_{ni}^{\circ}(\cdot,\cdot)$ does not depend on good j. It still depends, however, on source country i and the destination country n. This can be attributed

[6]We use the convention that realization of random variables are denoted in lowercase letters whereas random variables are denoted in uppercase.

to factors such as transportation costs between source country and destination country and input costs in source country. The specification of marginal cost is similar to Eaton and Kortum (2002) as it is multiplicatively seperable in z_{ni}^j, though it does not have to be constant in our case. We will not make any further functional form assumption on marginal cost.[7] Hereafter, we will call $c_{ni}^{\circ}(\cdot,\cdot)$ the base marginal cost.

The total demand for imports of good j in destination n is denoted by Q_n^j.

Assumption A3 :

(i) For any given n, Q_n^j is drawn independently from $F_{Q_n}(\cdot|b^j)$ for each j where $F_{Q_n}(\cdot|b^j)$ is absolutely continuous with strictly positive density, $f_{Q_n}(\cdot|b^j) > 0$, over its support, $\mathcal{S}_{Qn} = [0, \infty)$.

(ii) For all n and j, $\left\{Z_{n1}^j, ., Z_{ni}^j, ., Z_{nN}^j; i \neq n\right\}_{j=1}^J$ and $\{Q_n^j\}_{j=1}^J$ are independent given b^j.

Note that given the good characteristics distribution $F_{Q_n}(\cdot|\cdot)$ does not depend on good j. However, it depends on destination country n which might be attributed to factors such as income, special tastes of destination country n. Also, Assumption A3 states that given good characteristics, demand for imports is independent of the productivities of the exporter countries. One implication is that the demand in the market for imports does not depend on the clearing price in the import market. The theoretical model below can be extended to the general case where Q_n^j is a function of the clearing price in the import market and some demand shock. For the time being, however, our identification results are for the special case where Q_n^j is exogenous.

In our model, countries can only observe their own productivity but not the others', in other words, z_{ni}^j is private information. The productivity distributions, $F_{ni}(\cdot)$'s, however, are common knowledge to all countries in the sense that even though countries do not know what the actual productivities of other countries are; they have at least some idea about the

[7] In Eaton and Kortum (2002) and others we also see Samuelson's iceberg transportation costs assumption which is a special case of transportation costs entering marginal cost multiplicatively. The idea is that in order to supply one unit of any good j in country n, country i needs to produce $d_{ni} > 1$ units of good j since $(d_{ni} - 1) > 0$ "melts away" while being transported.

productivities of the others'. Exporting countries make their decisions about their supply before demand for imports is realized.

In the presence of uncertainty, as Klemperer and Meyer (1989) argue, agents might not want to commit to a fixed price or a fixed quantity and all decisions cannot be deferred until the uncertainty is resolved. For instance decisions about size and structure of an industry that produces a certain good have to be made well in advance. According to Klemperer and Meyer (1989) such decisions implicitly determine a supply function relating the quantity that will be sold at any given price. They further argue that instead of committing to a fixed price or a fixed quantity, committing to supply functions, will provide higher expected profits since it allows better adaptation to uncertainty. In our model for any country i, there are actually two sources of uncertainty: One is due to the uncertain demand for imports and the second is due to the unknown productivities of the other countries. Thus, given this uncertain environment, countries find it their best interest to offer supply functions instead of a fixed price or a fixed quantity.

Our model, however, differs from the supply function competition model of Klemperer and Meyer (1989) since in their model there is complete information about costs whereas in our case there is incomplete information about productivities. Even with the advanced information technologies of today, monitoring the behavior of all the rival countries in the production of every single good or in every industry would be extremely costly and still might not be possible. We follow Wilson (1979) to incorporate incomplete information.

Each exporter country i is offering a supply function (schedule) which shows at every price what country i is willing to supply of good j. We denote the supply schedule of exporter country i for destination n for good j by $s^j_{ni}(p, z^j_{ni})$. This supply function naturally depends on country i's productivity z^j_{ni}, which is private information, but not the productivities of the others, $z^j_{ni', i' \neq n}$. Once their productivities are realized and before the realization of demand for imports of good j, countries decide on their supply schedules.[8] These individual supply

[8]For each market n, country i solves an independent problem in the sense that we do not deal with an allocation across markets problem.

schedules are horizontally added to get aggregate supply and once demand is realized the market clearing price is determined where this aggregate supply intersects aggregate demand. The intersection of the market clearing price with each individual supply schedule gives the market clearing quantity supplied by each seller.

Denote the market clearing price in market n for good j by $P_n^{c,j}$. Define the quantity supplied by country i at the market clearing price as $Q_{ni}^j = s_{ni}^j(P_n^{c,j}, Z_{ni}^j)$. Note that by definition $\sum_{i \neq n} Q_{ni}^j = Q_n^j$. Define the total quantity supplied by all countries but i at the market clearing price by $Q_{n,-i}^j$, where $Q_{n,-i}^j = \sum_{k \neq i,n} Q_{nk}^j$. Also, denote the revenue of country i at the market clearing price by X_{ni}^j. An important feature of bilateral trade data is that even at the most disaggregated level, within the same product category unit values vary across exporters, i.e. $X_{ni}^j / Q_{ni}^j \neq X_{ni'}^j / Q_{ni'}^j$ for all $i \neq i', n$. This suggests discriminatory pricing rather than uniform pricing. In the former case, countries can charge different prices for different units of the same good, whereas they charge the same price for every unit in the latter. Hortacsu and Puller (2008) and Vives (2008) study supply function competition with private information under uniform pricing.

In the discriminatory case, the revenue of country i in country n for good j or expenditure of country n on good j from country i will be the area under the supply schedule upto the equilibrium quantity, i.e.

$$X_{ni}^j = \int_0^{s_{ni}^j(P_n^{c,j}, Z_{ni}^j)} {s_{ni}^j}^{-1}(q, Z_{ni}^j)\ dq = \int_0^{Q_{ni}^j} {s_{ni}^j}^{-1}(q, Z_{ni}^j)\ dq$$

where ${s_{ni}^j}^{-1}(q, Z_{ni}^j)$ is the inverse of $s_{ni}^j(p, Z_{ni}^j)$ with respect to the first argument, i.e., ${s_{ni}^j}^{-1}(q, Z_{ni}^j) = p$.

Notice that this environment is very similar to the divisible good auction (share auction) environment introduced by Wilson (1979) in which buyers instead of sellers compete. In a share auction there is a divisible item (or identical units of the same item) and each bidder submits a bid schedule (demand schedule) which shows at every price how many units (what share) she wants to have. This underlying marginal valuation determines the demand schedule the bidder offers, just as the underlying marginal cost determines the supply schedule in our model. Some part or all of this marginal valuation can be private information in the sense that bidders only know about their own but not the others'. This is again analogous to our case where productivities, which are part of marginal cost, are private information. Market clearing price is determined in a similar fashion: Individual bid schedules are horizontally added to get aggregate demand and the market clearing price is where this aggregate demand meets aggregate supply. The intersection of the market clearing price with the individual bid function gives the number of units won by that bidder and in the case of discriminatory pricing the area below the individual bid function up to this quantity will be the amount she has to pay. Hortacsu (2002) studies discriminatory share auction in his analysis of Turkish Treasury Bill Auctions wheras Hortacsu and Kastl (2008), Kastl (2010) consider uniform pricing.

For notational simplicity hereafter, we will drop hereafter index j and the conditioning on its characteristics b^j, as the analysis is performed for a given good j. Thus, market clearing price P_n^c is determined by:

$$\sum_{i \neq n} s_{ni}(P_n^c, Z_{ni}) = Q_n \tag{1}$$

which states that at the market clearing price, P_n^c, total supply of exporters should match the import demand in market n. Following Wilson (1979) and Hortacsu (2002), we define the probability distribution function of the market clearing price P_n^c in market n, from the point of view of country i as:

$$\begin{aligned}
\widetilde{H}_{ni}(p_n|z_{ni}) &= \Pr[P_n^c \leq p_n | Z_{ni} = z_{ni}] && (2) \\
&= \Pr[s_{ni}(p_n, z_{ni}) + \sum_{k \neq i,n} s_{nk}(p_n, Z_{nk}) \geq Q_n | Z_{ni} = z_{ni}] \\
&\equiv H_{ni}(p_n, s_{ni}(p_n, z_{ni})) && (3)
\end{aligned}$$

This distribution will be of crucial importance in our analysis.

Now, we can define country i's problem for market n.[9] Given a particular realization of market clearing price, p_n^c, the profit of country i when it serves market n is:

$$\pi(s_{ni}(p_n^c, z_{ni})) = \int_0^{s_{ni}(p_n^c, z_{ni})} \left[s_{ni}^{-1}(q, z_{ni}) - c_{ni}(q, z_{ni})\right] \, dq \tag{4}$$

That is the profit integrates the difference between the price and marginal cost for every quantity up to market clearing quantity $s_{ni}(p_n^c, z_{ni})$. This, however, is profit at a particular p_n^c. Since p_n^c can take any value in the support $[\underline{p}_n, \overline{p}_n]$, country i's expected profit maximization problem for market n is:

$$\max_{s_{ni}(\cdot, z_{ni})} \int_{\underline{p}_n}^{\overline{p}_n} \underbrace{\left\{ \int_0^{s_{ni}(p_n^c, z_{ni})} \left[s_{ni}^{-1}(q, z_{ni}) - c_{ni}(q, z_{ni})\right] \, dq \right\}}_{\pi(s_{ni}(p_n^c, z_{ni}))} \underbrace{d\widetilde{H}_{ni}(p_n^c | z_{ni})}_{dH_{ni}(p_n^c, s_{ni}(p_n^c, z_{ni}))} \tag{5}$$

Country i's strategy is its supply function $s_{ni}(\cdot, \cdot)$. Using calculus of variations, the necessary condition for this functional optimization problem which defines country i's optimal supply schedule in a Bayesian Nash equilibrium is given by:

$$p_n = c_{ni}(s_{ni}(p_n, z_{ni}), z_{ni}) + \frac{1 - H_{ni}(p_n, s_{ni}(p_n, z_{ni}))}{H_{ni,p}(p_n, s_{ni}(p_n, z_{ni}))} \tag{6}$$

for all p_n in the support, where $H_{ni,p}(\cdot, \cdot)$ is the (partial) derivative of $H_{ni}(\cdot, \cdot)$ with respect to the first argument. See also the Appendix. Plugging in our marginal cost specification, (6) becomes

$$p_n = \frac{c_{ni}^{\circ}(s_{ni}(p_n, z_{ni}))}{z_{ni}} + \frac{1 - H_{ni}(p_n, s_{ni}(p_n, z_{ni}))}{H_{ni,p}(p_n, s_{ni}(p_n, z_{ni}))} \tag{7}$$

for all p_n in its support. The necessary condition (7) above has an intuitive meaning. It says that the optimum price that country i charges for the $s_{ni}(p_n, z_{ni})$th unit in market n is the sum of marginal cost of that unit and some markup since $\frac{1-H_{ni}(p_n, s_{ni}(p_n, z_{ni}))}{H_{ni,p}(p_n, s_{ni}(p_n, z_{ni}))} \geq 0$.[10] Note

[9] For any market n, country i solves an independent problem in the sense that we do not deal with an allocation across markets problem.

[10] Reny (1999) shows the existence of a Bayesian Nash Equilibrium in pure strategies in which bidders

that this markup is not necessarily constant across units or exporters or destinations.

For identification purposes, which is the topic of the next section; we need a normalization to separate the effect of $c^{\circ}_{ni}(\cdot)$ and z_{ni}.

Assumption A4 : *For all n and i, $\overline{z}_{ni}$ is normalized to be $\overline{z} = 1$.*

In this case the base marginal cost function $c^{\circ}_{ni}(\cdot)$ can also be viewed as the marginal cost frontier of country i exporting to country n.

Lastly, we make the following assumption:

Assumption A5 : *For all n,i and j, $Q^j_{ni} = s^j_{ni}(P^{c,j}_n, Z^j_{ni}) > 0$.*

This assumption ensures that at equilibrium, every country has a strictly positive supply of good j in every market n.[11] It is important to mention that this assumption is required only for nonparametric identification which is the topic of the next section.

3 Nonparametric Identification

In this section we investigate whether we can recover uniquely the structure of the model from the observables. The structure we are after are the productivity distribution and the base marginal cost $[F_{ni}(\cdot), c^{\circ}_{ni}(\cdot)]$ for each n and i. The observables are the bilateral trade quantities between countries and the market clearing price $\left\{Q^j_{ni}, (P^c)^j_n\right\}_{j=1}^J$ for each n and i as well as the good characteristics $\left\{b^j\right\}_{j=1}^J$. Hence, the problem of identification is whether we can recover these unobservable productivity distributions and costs using observables from the data.

Bilateral trade data usually consists of bilateral trade quantities and expenditures between countries$\left\{Q^j_{ni}, X^j_{ni}\right\}_{j=1}^J$ for each n and i. Pehlivan and Vuong (2010b) show how to obtain market clearing price under discriminatory pricing when only revenue and quantity

employ nondecreasing functions for the asymmetric discriminatory share auction.

[11] Note that this is an assumption on the equilibrium quantity and not on the primitives. A case when it is satisfied is when $c^{\circ}_{ni}(0) = c_o$, $c_o \geq 0$ across $i \neq n$. It, however, can be also satisfied in other cases.

data are available.[12] We follow that approach to obtain market clearing prices $\{(P^c)_n^j\}_{j=1}^J$ for each n as is explained in detail in Section 4.3.

It is important to note that unlike Hortacsu (2002), Kastl (2010) and others, we do not observe the whole price-quantity schedules of players but observe *only the transaction/equilibrium points*. Basically, for any given n, i, and j only *one point* on the supply schedule is observed. The identification strategy proposed in this section, however, identifies the structure when only the transaction points are observed.

Our identification strategy basically follows Guerre, Perrigne and Vuong (2000) where they use the necessary condition of a first-price auction which characterizes the optimal bid function to identify the distribution of valuations in the case of a first-price auction. In their case, bids can be observed but the underlying private valuations cannot. In our case prices can be observed but the underlying productivities cannot. They use the necessary condition which characterizes the optimal bid and rewrite it in a way that everything except the private valuation term can be expressed in terms of the observables (either the bid itself, distribution of bids or density of bids which are observed from data). What they call *pseudo-valuations* can, then, be obtained and hence their distribution. In our case, the idea is very similar as we express the necessary condition (7) in terms of the observables, with two differences: First, obtaining the distribution of market clearing price from i's point of view is not as trivial as obtaining the distribution of bids and second, we also need to obtain the marginal cost from the observables before obtaining what we can call *pseudo-productivities*.

In this section, for any good j, we take the joint distribution $F_{Q_{ni}^j,(Q_{nk}^j)_{k\neq i,n},Q_n^j,P_n^{c,j}}(\cdot,...,\cdot)$ of the observables as known. Hence, all the marginals and conditionals are known. In Section 5, we discuss how such a distribution can be estimated from bilateral trade data.[13]

[12] They basically use the simple fact that the market clearing price is the derivative of the revenue with respect to the quantity in discriminatory pricing.

[13] The fact that our observations are coming from a cross section and that we have only one observation for good j might be confusing at this stage. For now, it might be helpful to think we have many observations for good j such as in a time series fashion and that is how the joint distribution of the observables is obtained.

Dropping again the index j and the conditioning on b^j, identification is performed for a given good j, we proceed as follows: We first identify $H_{ni}(\cdot,\cdot)$ and then we identify $c^{\circ}_{ni}(\cdot)$. Lastly, we will identify $F_{ni}(\cdot)$.

3.1 Identification of $H_{ni}(\cdot,\cdot)$

For each i, define the random variable $S_{ni,p_n} = s_{ni}(p_n, Z_{ni})$ for any arbitrary p_n. By Assumption A1, S_{ni,p_n}'s are also mutually independent across i. Note that this p_n is a generic price, not the market clearing price. If it were market clearing price, then they would not be independent. Define the total quantity supplied by all exporters, $i \neq n$, at an arbitrary p_n as $\Sigma_{n,p_n} = \sum_{i\neq n} s_{ni}(p_n, Z_{ni})$, the total quantity supplied by all the other exporters but i at an arbitrary p_n as $\Sigma_{n,-i,p_n} = \sum_{k\neq n,i} s_{nk}(p_n, Z_{nk})$. Also, define $XD_{n,p_n} = Q_n - \sum_{i\neq n} s_{ni}(p_n, Z_{ni})$ which is the excess demand for any arbitrary p_n.

Let $q_{ni} = s_{ni}(p_n, z_{ni})$. Using (2) and by Assumption A1 and A3 we get[14]

$$H_{ni}(p_n, s_{ni}(p_n, z_{ni})) = \int_0^\infty \Pr[\Sigma_{n,-i,p_n} \geq q_n - q_{ni}] f_{Q_n}(q_n) dq_n \tag{8}$$

Define $f_{\Sigma_{n,p_n}}(\cdot)$ as the density of Σ_{n,p_n} and define $f_{\Sigma_{n,-i,p_n}}(\cdot)$ as the density of $\Sigma_{n,-i,p_n}$ for all i and for all n. Thus,

$$H_{ni}(p_n, q_{ni}) = \int_0^\infty \left[\int_{q_n - q_{ni}}^\infty f_{\Sigma_{n,-i,p_n}}(q_{n,-i}) dq_{n,-i} \right] f_{Q_n}(q_n) dq_n \tag{9}$$

Noting that $f_{Q_n}(\cdot)$ is observed from the data, if we can get $f_{\Sigma_{n,-i,p_n}}(\cdot)$ from the observables, then we can identify $H_{ni}(p_n, q_{ni})$ for all (p_n, q_{ni}). In order to express $f_{\Sigma_{n,-i,p_n}}(\cdot)$ in terms of observables consider the following relationships:

[14] We obtain this using $H_{ni}(p_n, s_{ni}(p_n, z_{ni})) = \Pr[P^c_n \leq p_n | Z_{ni} = z_{ni}] = \Pr[S_{ni,p_n} + \Sigma_{n,-i,p_n} \geq Q_n | Z_{ni} = z_{ni}] = \Pr[q_{ni} + \Sigma_{n,-i,p_n} \geq Q_n | Z_{ni} = z_{ni}] = \Pr[\Sigma_{n,-i,p_n} \geq Q_n - q_{ni}] = \int_0^\infty \Pr[\Sigma_{n,-i,p_n} \geq q_n - q_{ni} | Q_n = q_n] f_{Q_n}(q_n) dq_n = \int_0^\infty \Pr[\Sigma_{n,-i,p_n} \geq q_n - q_{ni}] f_{Q_n}(q_n) dq_n$

First, note that $\Pr\left[Q_n \leq q_n | P_n^c = p_n\right] = \Pr\left[\Sigma_{n,p_n} \leq q_n | XD_{n,p_n} = 0\right]$.[15] By using Assumption A3 and relevant transformations, we get for any (p_n, q_n):

$$f_{\Sigma_{n,p_n}}(q_n) = \frac{f_{P_n^c|Q_n}(p_n|q_n)}{\int_0^\infty f_{P_n^c|Q_n}(p_n|\widetilde{q_n})\, d\widetilde{q_n}} \tag{10}$$

See Appendix. Using Bayes' Rule, (10) can be written as

$$f_{\Sigma_{n,p_n}}(q_n) = \frac{f_{Q_n|P_n^c}(q_n|p_n)}{f_{Q_n}(q_n) \int_0^\infty \frac{f_{Q_n|P_n^c}(\widetilde{q_n}|p_n)}{f_{Q_n}(\widetilde{q_n})} d\widetilde{q_n}} \tag{11}$$

Thus, (11) can be interpreted as follows: What would be our initial guess for the density of total supply given some p_n ? At first glance, one may think it is $f_{Q_n|P_n^c}(q_n|p_n)$. This would be incorrect because we are looking for the density of total supply at some arbitrary p_n which is not necessarily the market clearing price for that particular total demand $Q_n = q_n$. In other words, the total supply Σ_{n,p_n} might not match the total demand Q_n at that arbitrary p_n. Consequently, (Σ_{n,p_n}, p_n) might not be the equilibrium pair. In $f_{Q_n|P_n^c}(q_n|p_n)$, however, p_n is the market clearing price for that particular total demand Q_n. The expression in the denominator of (11) is actually the term that corrects our initial guess for the fact that p_n might not necessarily be the market clearing price when $Q_n = q_n$. On the other hand, note that $f_{Q_n|P_n^c}(q_n|p_n) = f_{\Sigma_{n,p_n}|P_n^c}(q_n|p_n)$ which is the conditional density of total supply given market clearing price but we are looking for the marginal density.

Second, we note that $\Pr[Q_{n,-i} \leq q_{n,-i} | P_n^c = p_n,\ Q_n = q_n] = \Pr[\Sigma_{n,-i,p_n} \leq q_{n,-i} | \Sigma_{n,p_n} = q_n, Q_n = q_n]$.[16] Similarly, using Assumption A3, A5 and relevant transformations, we get for

[15]This follows from $\Pr[Q_n \leq q_n | P_n^c = p_n] = \Pr[Q_n \leq q_n | \Sigma_{n,p_n} = Q_n] = \Pr[\Sigma_{n,p_n} \leq q_n | XD_{p_n} = 0]$.

[16]This follows from $\Pr[Q_{n,-i} \leq q_{n,-i} | P_n^c = p_n,\ Q_n = q_n] = \Pr[\Sigma_{n,-i,p_n} \leq q_{n,-i} | P_n^c = p_n,\ Q_n = q_n] = \Pr[\Sigma_{n,-i,p_n} \leq q_{n,-i} | \Sigma_{n,p_n} = q_n,\ Q_n = q_n]$.

any $(p_n, q_{n,-i})$:[17]

$$f_{\Sigma_{n,-i,p_n}}(q_{n,-i}) = \int_0^\infty f_{\Sigma_{n,p_n}}(q_n) f_{Q_{n,-i}|P_n^c,Q_n}(q_{n,-i}|p_n, q_n)\, dq_n \tag{12}$$

See Appendix. Equation (12) can be interpreted in a similar fashion. What would be our initial guess for the density of total quantity supplied by the others, $\Sigma_{n,-i,p_n,}$ given some p_n? A first but incorrect guess would be:

$$f_{Q_{n,-i}|P_n^c}(q_{n,-i}|p_n) = \int_0^\infty f_{Q_n|P_n^c}(q_n|p_n)\, f_{Q_{n,-i}|P_n^c,Q_n}(q_{n,-i}|p_n, q_n)\, dq_n. \tag{13}$$

When we substitute for (11) for $f_{\Sigma_{n,p_n}}(q_n)$ into (12), we have

$$f_{\Sigma_{n,-i,p_n}}(q_{n,-i}) = \int_0^\infty \frac{f_{Q_n|P_n^c}(q_n|p_n)\, f_{Q_{n,-i}|P_n^c,Q_n}(q_{n,-i}|p_n, q_n)}{f_{Q_n}(q_n) \int_0^\infty \frac{f_{Q_n|P_n^c}(\widetilde{q_n}|p_n)}{f_{Q_n}(\widetilde{q_n})} d\widetilde{q_n}} dq_n \tag{14}$$

Note that the weights used in (13) are $f_{Q_n|P_n^c}(q_n|p_n)$ whereas the weights used in (14) are $\frac{f_{Q_n|P_n^c}(q_n|p_n)}{f_{Q_n}(q_n) \int_0^\infty \frac{f_{Q_n|P_n^c}(\widetilde{q_n}|p_n)}{f_{Q_n}(\widetilde{q_n})} d\widetilde{q_n}}$ which correct $f_{Q_n|P_n^c}(q_n|p_n)$ similarly as in (11). Again this is due to the fact that we would like to know the density of $\Sigma_{n,-i,p_n}$, at an arbitrary p_n.

The next lemma provides an alternative expression for (14) which is more suitable for estimation:

Lemma 1: *Under Assumption A1, A3 and A5, the density $f_{\Sigma_{n,-i,p_n}}(\cdot)$ of $\Sigma_{n,-i,p_n}$, is identified by*

$$f_{\Sigma_{n,-i,p_n}}(q_{n,-i}) = \frac{\int_0^\infty f_{Q_{n,-i},P_n^c|Q_n}(q_{n,-i}, p_n|q_n)\, dq_n}{\int_0^\infty f_{P_n^c|Q_n}(p_n|q_n)\, dq_n} \tag{15}$$

for all $(p_n, q_{n,-i})$ and for all n and i.

[17] Unlike (10), however, (12) requires Assumption A5 which prevents any mass point in the distribution of $Q_{n,-i}$, a case we would have if some country is not serving market n.

Equation (15) follows from (14) by straight application of Bayes' Rule. Lemma 1 shows that the (marginal) density of $\Sigma_{n,-i,p_n}$, $f_{\Sigma_{n,-i,p_n}}(\cdot)$ is identified since everything on the right hand side can be estimated from the data.

Once we identify $f_{\Sigma_{n,-i,p_n}}(\cdot)$, everything on the right hand side of (9) is also known from the data. Hence, $H_{ni}(\cdot,\cdot)$ is identified. Moreover, its derivative $H_{ni,p}(\cdot,\cdot)$ is also identified. The next lemma states this result and gives an expression for this derivative.

Lemma 2: *The function $H_{ni}(\cdot,\cdot)$ is identified by (9) and (15) while its derivative $H_{ni,p}(\cdot,\cdot)$ is identified by*

$$H_{ni,p}(p_n, q_{ni}) = \int_0^\infty \left[\int_{q_n - q_{ni}}^\infty \frac{\partial \{ f_{\Sigma_{n,-i,p_n}}(q_{n,-i}) \}}{\partial p_n} dq_{n,-i} \right] f_{Q_n}(q_n) dq_n \tag{16}$$

for all (p_n, q_{ni}) and for all n and i.

.

Equation (16) follows from differentiating (9) with respect to the first argument.

3.2 Identification of $c^\circ_{ni}(\cdot)$

Denote the support of Q_{ni} by $\mathcal{S}_{Qni}$ for all n and for all i.

Proposition 1 : *Under Assumption A1-A5 and by necessary condition (7), $c^\circ_{ni}(\cdot)$ is identified on $S_{Qni} = [0, \overline{q}_{ni}]$, for all n and for all i.*

The identification argument is as follows: For a given market n, for a given i and for a given $q_o \in \mathcal{S}_{Qn}$, the support $\mathcal{S}_{Q_{ni},P^c_n|Q_n=q_o}$of (Q_{ni}, P^c_n) conditional on $Q_n = q_o$ is known. This corresponds to the area ABCD in Figure 1.[18] Define $\Sigma_{n,-i}(\cdot,\overline{z})$ to be total quantity supplied at any p_n by all exporters other than i, when all exporters other than i have the

[18] Without loss of generality, the strategies of countries are positioned as in Figure 1. Our identification argument, however, would work in any other case such as when strategies of countries intersect each other.

best productivity draws $\overline{z}$, i.e., $\Sigma_{n,-i}(\cdot,\overline{z}) = \sum_{k\neq n,i} s_{nk}(\cdot,\overline{z})$ and $\Sigma_{n,-i}^{-1}(\cdot,\overline{z})$ is the inverse of it with respect to the first argument. Similarly, define $\underline{z} = (\underline{z}_{nk})_{k\neq n,i}$. The curves with tildas in Figure 1, e.g., $\widetilde{\Sigma}_{n,-i}^{-1}(\cdot,\overline{z},q_o)$, are the symmetries of the original ones, e.g., $\Sigma_{ni}^{-1}(\cdot,\overline{z})$, with respect to $q_o/2$. Hence, $\widetilde{\Sigma}_{n,-i}^{-1}(\cdot,\overline{z},q_o)$ can be thought of as the residual demand for i, when all her rivals have the best productivity draws. It is important to note that the optimal strategies will not depend on the realized value q_o of Q_n as Q_n is random by Assumption A3-(i), the residual demands, however, will depend on q_o.

Note that the minimum price, given $Q_n = q_o$, denoted $\underline{p}_n(q_o)$, is achieved at point A, since at point A all exporters have their best draws. Similarly, the maximum price, given $Q_n = q_o$, denoted $\overline{p}_n(q_o)$, is achieved at point C, since at point C, all countries have their worst draws. In market n, the minimum quantity supplied by i, given $Q_n = q_o$, denoted $\underline{q}_{ni}(q_o)$, is achieved at point D, since at point D, country i has her worst draw while her rivals all have their best draws. By a similar argument, in market n, the maximum quantity supplied by i, given $Q_n = q_o$, denoted $\overline{q}_{ni}(q_o)$, is achieved at point B.

As the optimal strategy will not depend on the realized value q_o of Q_n since Q_n is random by Assumption A3-(i), due to variations in Q_n, the support $\mathcal{S}_{Q_{ni},P_n^c|Q_n=q_o}$ of (Q_{ni},P_n^c) conditional on Q_n (area ABCD in Figure 1) will move to cover an area between the supply schedules $s_{ni}^{-1}(\cdot,\underline{z})$ and $s_{ni}^{-1}(\cdot,\overline{z})$, thereby providing the unconditional support $\mathcal{S}_{Q_{ni},P_n^c}$. As $\mathcal{S}_{Qn} = [0,\infty)$ by Assumption A3-(i) the other two boundaries of the unconditional support $\mathcal{S}_{Q_{ni},P_n^c}$ of (Q_{ni},P_n^c) are given by $Q_{ni} = 0$ and the highest possible residual demand, $\sup_{q_o\in[0,\infty)} \widetilde{\Sigma}_{n,-i}^{-1}(\cdot,\underline{z},q_o)$. In particular, the support $\mathcal{S}_{Qni}$ of Q_{ni} is $[0, \sup_{q_o\in[0,\infty)} \overline{q}_{ni}(q_o)]$.

Since the support $\mathcal{S}_{Q_{ni},P_n^c}$ is known, its lower boundary is also known, hence $s_{ni}^{-1}(\cdot,\overline{z})$ is known on $\mathcal{S}_{Qni}$. By the necessary condition (7) and Assumption A4 which sets $\overline{z}=1$, $c_{ni}^{\circ}(\cdot)$ is identified over $\mathcal{S}_{Qni}$ by

$$c_{ni}^{\circ}(\cdot) = s_{ni}^{-1}(\cdot,1) - \frac{1 - H_{ni}(s_{ni}^{-1}(\cdot,1),\cdot)}{H_{ni,p}(s_{ni}^{-1}(\cdot,1),\cdot)} \tag{17}$$

where $H_{ni}(\cdot,\cdot)$ and $H_{ni,p}(\cdot,\cdot)$ are identified by Lemma 2.

3.3 Identification of $F_{ni}(\cdot)$

The next proposition establishes identification of the productivity distribution $F_{ni}(\cdot)$ for any n and for any i.

Proposition 2 : *Under Assumption A1-A5 and by necessary condition (7), $F_{ni}(\cdot)$ and $\underline{z}_{ni}$ are identified for all n and for all i.*

Once we identify $c^{\circ}_{ni}(\cdot)$, now we know the base marginal cost for any possible q_{ni}. Hence, for any (q_{ni}, p^c_n) observation, we can now use our necessary condition to recover z_{ni}. Namely from (7) we have:

$$p^c_n = \frac{c^{\circ}_{ni}(q_{ni})}{z_{ni}} + \frac{1 - H_{ni}(p^c_n, q_{ni})}{H_{ni,p}(p^c_n, q_{ni})}$$

which gives

$$z_{ni} = \frac{c^{\circ}_{ni}(q_{ni})}{p^c_n - \frac{1-H_{ni}(p^c_n, q_{ni})}{H_{ni,p}(p^c_n, q_{ni})}} \tag{18}$$

where everything on the right hand side of (18) is known. Once we know z_{ni}'s their distribution is also identified. Thus, $F_{ni}(\cdot)$ is identified.

In particular, the lower bound $\underline{z}_{ni}$ is identified by

$$\underline{z}_{ni} = \frac{c^{\circ}_{ni}(\cdot)}{s^{-1}_{ni}(\cdot, \underline{z}) - \frac{1-H_{ni}(s^{-1}_{ni}(\cdot,\underline{z}),\cdot)}{H_{ni,p}(s^{-1}_{ni}(\cdot,\underline{z}),\cdot)}} \tag{19}$$

4 Data

In this section we first describe our data. We, then, introduce a hypothetical good to deal with the cross section nature of the data and in particular, the heterogenity across goods and the difference in measurement units. The last section deals with the construction of the market clearing price.

4.1 Data description

Our source of trade data is the United Nations Commodity Trade Statistics Database (COMTRADE) which is accessible online. As in Eaton and Kortum (2002) we use bilateral trade data for manufacturing imports in 1990. Bilateral trade data consists of traded quantity and trade values (in dollars) between countries and is available on disaggragated level according to different classifications. The disaggregation level is indicated by digits and higher digits correspond to more disaggregated product categories. Eaton and Kortum (2002) look at 19 OECD countries and use 4-digit SITC (Standard International Trade Classification) Revision2 data.[19] To determine which product categories correspond to the Bureau of Economic Analysis (BEA) manufacturing industry codes they use Maskus (1991) concordance. They, then aggregate to obtain total manufacturing imports between countries. In contrast to Eaton and Kortum (2002), we use the data at the disaggregated level, in particular, 5-digit SITC Revision 2, and the categories corresponding to BEA manufacturing industries according to the concordance provided by Feenstra, Lipsey and Bowen (1997).[20,21]

For this particular application we look at the German market, i.e. n = Germany.[22]. Our sample consists of the 7 largest (in value) exporters listed in Table 1. Out of 19 countries in the Eaton and Kortum (2002) sample, the value of manufacturing imports from these 7 exporters constitute almost 80% of value of total manufacturing imports of Germany as shown in Table 1. Also, in our sample, the total value of manufacturing imports in product categories where we observe zeros (cases when at least one country is not exporting to

[19] Those 19 countries are: Australia, Austria, Belgium, Canada, Denmark, Finland, France, Germany, Greece, Italy, Japan, Netherlands, New Zealand, Norway, Portugal, Spain, Sweden, United Kingdom, United States. Also, Eaton and Kortum (2002) uses United Nations-Statistics Canada data (World Trade Database) as explained by Feenstra, Lipsey and Bowen (1997) which is a slightly modified version of United Nations Commodity Trade Statistics Data.

[20] Whenever 5 digit level is available we use data at this level. For some categories no further classification was available after 4 digits. In those cases we use the 4 digit level.

[21] Eaton and Kortum (2002) mention that using the concordance by Feenstra, Lipsey and Bowen (1997) made no difference in their case.

[22] This is actually the Federal Republic of Germany according to the United Nations Commodity Trade Statistics Database. Also, Belgium in their sample is actually Belgium and Luxembourg since they were still reporting together. We, however, follow the convention in Eaton and Kortum (2002) and call them Germany and Belgium, respectively.

Germany), constitute only 9% of the total value of manufacturing imports in our sample. When we drop those, we have $J = 969$ products where all the exporters in our sample export some positive amounts. Hence, we have $\left\{Q_{ni}^{j}, X_{ni}^{j}\right\}_{j=1}^{J}$, where $n =$ Germany, $i = 1, ..., 7$ and $J = 969$.

One important thing to notice is that within a given product category j, the unit values X_{ni}^{j}/Q_{ni}^{j} vary across i, which justifies our discriminatory pricing assumption. Table 2 provides summary statistics for the coefficient of variation of unit values across exporters within a given product category. The mean is 0.67 indicating that the standard deviation of unit values across exporters within a given product category is more than half of the mean of unit values across exporters in that category. This indicates that there is significant amount of variations in unit values across exporters on average.

Another issue with this data is that trade quantities and unit prices depend on the unit of measurement. Some goods are measured in kilograms, some are measured in liters, some are measured in units, etc. Therefore, when units of measurement are changed, the same information can be represented in totally different quantities and unit prices. Obviously, this is not going to be the case for trade value since it is in dollars. The next section explains how we deal with this issue.

4.2 Introducing Hypothetical Good A

The observed bilateral trade quantities and the unit prices depend on the measurement unit of each good. To control for the differences in measurement units, we introduce a hypothetical good A. The idea is to express all data that depend on measurement units in terms of this hypothetical good A, so that we have repetitions of the same game for this hypothetical good A.[23]

For any country i and destination n, let the total cost of producing q units of good

[23] If we had many observations on the same good such as in a time series situation or many observations for different goods having the same unit of measurement and similar characteristics, we would not need to consider this hypothetical good and make the assumptions below. Conditioning on b^j would be sufficient.

j be given by $TC^j_{ni}(q, b^j, z^j_{ni})$ and the total cost of producing a units of the hypothetical good A at the productivity level realization z^j_{ni} be given by $TC^A_{ni}(a, z^j_{ni})$. Let $\frac{\partial TC^j_{ni}(q,b^j,z^j_{ni})}{\partial q} = c^j_{ni}(q, b^j, z^j_{ni})$ and $\frac{\partial TC^A_{ni}(a,z^j_{ni})}{\partial a} = c^A_{ni}(a, z^j_{ni})$ where $c^A_{ni}(a, z^j_{ni})$ is the marginal cost of producing the ath unit of hypothetical good A at productivity level realization z^j_{ni}. Following Assumption A2-A3, we make the following assumptions:

Assumption A2′ :

(i) For any good j and some $\lambda^j > 0$, $TC^j_{ni}(q, b^j, z^j_{ni}) = TC^A_{ni}(\lambda^j q, z^j_{ni})$ where $TC^A_{ni}(\cdot, \cdot)$ is twice continuously differentiable in a, for all $a \geq 0$, for all n and for all i.[24]

(ii) For all $a \geq 0$ and for all n, i and j, $c^A_{ni}(a, z^j_{ni}) = \frac{c^{\circ,A}_{ni}(a)}{z^j_{ni}}$ where $c^{\circ,A}_{ni}(\cdot)$ is weakly increasing in a.

Assumption A2′-(i) says that given any destination n and exporter i, for any good j the total cost of producing q^j_{ni} units of good j is the total cost of producing $\lambda^j q^j_{ni}$ units of the hypothetical good A. We denote that amount by A^j_{ni}, so $A^j_{ni} = \lambda^j Q^j_{ni}$. For any good j, A^j_{ni} may be interpreted as the quantity of the hypothetical good A that is cost equivalent to the quantity Q^j_{ni} of good j. Similarly, we can define $A^j_n = \lambda^j Q^j_n$ where Q^j_n was defined to be the total demand for imports in destination n for good j. Thus, A^j_n is the amount of total demand in terms of good A. An immediate implication of Assumption A2′ is that $c^\circ_{ni}(q, b^j) = \lambda^j c^{\circ,A}_{ni}(\lambda^j q)$.[25]

Assumption A3′ :

(i) *For any given n, A^j_n is drawn independently from $F_{A_n}(\cdot)$ for each j where $F_{A_n}(\cdot)$ is absolutely continuous with density $f_{A_n}(\cdot) > 0$ over the support $\mathcal{S}_{A_n}[0, \infty)$.*

(ii) *For all n and j, $\left\{Z^j_{n1}, .., Z^j_{ni}, .., Z^j_{nN}; i \neq n\right\}^J_{j=1}$ and A^j_n are independent.*

A consequence of Assumption A3′, is that $F_{Q_n}(q|b^j) = F_{A_n}(\lambda^j q)$ for all $q \geq 0$.

[24] More generally, we could have $TC^j_{ni}(q, b^j, z^j_{ni}) = TC^A_{ni}(\lambda^j q, z^j_{ni}) + \kappa^j$ instead of A2′-(i). This will not change our results.

[25] Since $TC^j_{ni}(q, b^j, z^j_{ni}) = TC^A_{ni}(\lambda^j q, z^j_{ni})$, taking derivative with respect to q of both sides yields $c^j_{ni}(q, b^j, z^j_{ni}) = \lambda^j c^A_{ni}(\lambda^j q, z^j_{ni})$. Thus, $c^\circ_{ni}(q) = \lambda^j c^{\circ,A}_{ni}(\lambda^j q)$.

Introducing this hypothetical good does not change the nature of the solution in our model as well as our identification argument. The only difference is that the quantities are expressed in terms of the hypothetical good A. Recall that the supply schedule of the exporter country i for destination n and good j was denoted by $s^j_{ni}(\cdot, z^j_{ni})$. Now denote the supply schedule of exporter country i for destination n for good A for the same level of productivity realization z^j_{ni} by $s^A_{ni}(\cdot, z^j_{ni})$. Under Assumption A2′-A3′ and the first order condition for each problem for good j, we have the following relation between the optimal schedules in terms of good j and hypothetical good A:

$$s^j_{ni}(p, z^j_{ni}) = \frac{1}{\lambda^j} s^A_{ni}(\frac{p}{\lambda^j}, z^j_{ni}) \tag{20}$$

for any p.

The market clearing price in destination n for good j was denoted by $P^{c,j}_n$. This market clearing price is a function of productivity of all exporters, the import demand and the good characteristics, i.e. $P^{c,j}_n = P^c_n(Z^j_{-n}, Q^j_n, b^j)$ where $Z^j_{-n} = (Z^j_{n1}, .., Z^j_{nn-1}, Z^j_{nn+1} .., Z^j_{nN})$. Now, let the market clearing price in destination n for good A at productivity Z_{-n} of all exporters where $Z_{-n} = (Z_1, .., Z_{nn-1}, Z_{nn+1,}.., Z_N)$ and import demand for A_n be given by $P^{c,A}_n = P^{c,A}_n(Z_{-n}, A_n)$. We, then, have the following relation between the market clearing price for good j and the hypothetical good A for any j:

$$P^{c,j}_n = \lambda^j P^{c,A,j}_n \tag{21}$$

where $P^{c,A,j}_n = P^{c,A}_n(Z^j_{n1}, .., Z^j_{ni}, .., Z^j_{nN}, \lambda^j Q^j_n)$. Let $A^j_{n,-i}$ be the total quantity of good A supplied by all countries but i, at the market clearing price $P^{c,A,j}_n$, i.e., $A^j_{n,-i} = \sum\limits_{k \neq i,n} s^A_{nk}(P^{c,A,j}_n, Z^j_{nk})$. We can replace any value in terms of good j with its good A analogue and all our analysis and results above follow. Note that expenditure in terms of good j, X^j_{ni}, and expenditure on the equivalent amount of good A are the same. Therefore, we will not differentiate between expenditures, it will be denoted by X^j_{ni} for both good j and good A.

Hereafter, we let $\lambda^j = \frac{\sum\limits_n \sum\limits_i X^j_{ni}}{\sum\limits_n Q^j_n}$, where λ^j can be considered as the average unit value of good j across all import markets around the world. Thus, for any j, λ^j is observed. Converting the supply Q^j_{ni} of any good j to the supply $A^j_{ni} = \lambda^j Q^j_{ni}$ of the hypothetical good A can be viewed as providing the value for Q^j_{ni} at an average world price.[26] Moreover, with such a choice of λ^j, we can control for heterogeneity across goods. Note that we could control for differences in measurement units and heterogeneity across goods by conditioning on the good characteristics including the unit of measurement. Using λ^j, we address both problems and save on conditioning variable(s). Table 3 provides some summary statistics for the hypothetical good A. The highest values for A, usually correspond to cars, more specifically "passenger motor vehicles excluding buses". We also see motor vehicles for transport of goods or materials and aircraft parts for some countries. The smallest A values correspond to goods such as pen nibs or matches.

4.3 Construction of $\left\{P^{c,A,j}_n\right\}^J_{j=1}$

As we mentioned before, usually in typical bilateral trade data we observe bilateral trade quantities and expenditures between countries$\left\{Q^j_{ni}, X^j_{ni}\right\}^J_{j=1}$ for each n and i but not the price levels. For our analysis we need the market clearing prices for each n, $\{P^{c,j}_n\}^J_{j=1}$Since in our estimation we use the values in terms of our hypothetical good A, we need to get $\left\{P^{c,A,j}_n\right\}^J_{j=1}$ for each n, using $\left\{A^j_{ni}, X^j_{ni}\right\}^J_{j=1}$ for each n and i.

Pehlivan and Vuong (2010b) show how to estimate market clearing price under discriminatory pricing when only revenue and quantity data are available. Basically they make use of the simple fact that the market clearing price is the derivative of the revenue with respect to the quantity in discriminatory pricing. They use Matzkin (2003) and extend it

[26] In our data, there are some markets in which goods in same categories are denominated in different units than other markets. Within the same market across exporters there is no difference, though, across markets there might be some difference. Thus, for the German market to calculate λ^j we use Austria, Belgium, Denmark, Finland, France, Germany, Greece, Italy, Netherlands, Portugal, Spain, Sweden and United Kingdom markets.

to the multidimensional error case using Hoderlein, Su and White (2010) to show that this derivative is identified.

First, note that expenditure $X_{ni}^{A,j} = \int_0^{A_{ni}^j} s_{ni}^{A\,-1}(a, Z_{ni}^j)da$ in terms of the hypothetical good A is related to trade value X_{ni}^j through $X_{ni}^{A,j} = X_{ni}^j$ for all n, i and j. Second, we note that $X_{ni}^j = x_{ni}(Z_{-n}^j, A_n^j)$ and $A_{ni}^j = a_{ni}(Z_{-n}^j, A_n^j)$ for any n, i, and j. Since $X_{ni}^j = \int_0^{A_{ni}^j} s_{ni}^{A\,-1}(a, Z_{ni}^j)\, da \equiv \psi_{ni}(A_{ni}^j, Z_{ni}^j)$, we have

$$P_n^{c,A,j} = s_{ni}^{A\,-1}(A_{ni}^j, Z_{ni}^j) = \frac{\partial X_{ni}^j}{\partial A_{ni}^j} = \frac{\partial \psi_{ni}(A_{ni}^j, Z_{ni}^j)}{\partial A_{ni}^j} \tag{22}$$

This derivative is hard to get since A_{ni}^j and Z_{ni}^j are not independent as they are linked through the equilibrium condition. Nevertheless, from the chain rule we can write:

$$\frac{\partial x_{ni}(Z_{-n}^j, A_n^j)}{\partial A_n} = \underbrace{\frac{\partial \psi_{ni}(A_{ni}^j, Z_{ni}^j)}{\partial A_{ni}^j}}_{P_n^{c,A,j}} \times \frac{\partial a_{ni}(Z_{-n}^j, A_n^j)}{\partial A_n} \tag{23}$$

Thus, for any n, i, and j, we have

$$P_n^{c,A,j} = \frac{\frac{\partial x_{ni}(Z_{-n}^j, A_n^j)}{\partial A_n}}{\frac{\partial a_{ni}(Z_{-n}^j, A_n^j)}{\partial A_n}} \tag{24}$$

Moreover, when we add this ratio across $i \neq n$

$$P_n^{c,A,j} = \frac{\sum\limits_{i\neq n} \frac{\partial x_{ni}(Z_{-n}^j, A_n^j)}{\partial A_n}}{\sum\limits_{i\neq n} \frac{\partial a_{ni}(Z_{-n}^j, A_n^j)}{\partial A_n}} = \frac{\frac{\partial \left(\sum\limits_{i\neq n} x_{ni}(Z_{-n}^j, A_n^j) \right)}{\partial A_n}}{\frac{\partial \left(\sum\limits_{i\neq n} a_{ni}(Z_{-n}^j, A_n^j) \right)}{\partial A_n}} = \frac{\frac{\partial x_n(Z_{-n}^j, A_n^j)}{\partial A_n}}{\frac{\partial a_n(Z_{-n}^j, A_n^j)}{\partial A_n}} = \frac{\partial x_n(Z_{-n}^j, A_n^j)}{\partial A_n} \tag{25}$$

where $X_n^j = x_n(Z_{-n}^j, A_n^j)$ and $A_n^j = a_n(Z_{-n}^j, A_n^j)$ for any n and j.[27]

[27] Note that the last equality in (25) follows from the fact that $\frac{\partial a_n(Z_{-n}^j, A_n^j)}{\partial A_n} = 1$.

Following Pehlivan and Vuong (2010b) we have

$$P_n^{c,A,j} = \frac{\partial F^{-1}_{X_n|A_n}(u_n^j|A_n^j)}{\partial A_n} \tag{26}$$

where $u_n^j = F_{X_n|A_n}(X_n^j|A_n^j)$. Since our data is highly skewed we use a logarithmic transformation of our variables. Define $LX_n^j = \ln(X_n^j)$, $LA_{ni}^j = \ln(A_{ni}^j)$, and $LA_n^j = \ln(A_n^j)$ for all n, i, and j. Using these transformations, equation (26) can be written as:

$$P_n^{c,A,j} = \frac{X_n^j}{A_n^j}\frac{\partial F^{-1}_{LX_n|LA_n}(u_n^j|LA_n^j)}{\partial LA_n} \tag{27}$$

where $u_n^j = F_{LX_n|LA_n}(LX_n^j|LA_n^j)$. Manipulating the derivative in (27) we get

$$\frac{\partial F^{-1}_{LX_n|LA_n}(u_n^j|LA_n^j)}{\partial LA_n} = \frac{\left[F_{LX_n|LA_n}(LX_n^j|LA_n^j)\frac{\partial f_{LA_n}(LA_n^j)}{\partial LA_n} - \frac{\partial}{\partial LA_n}\left\{F_{LX_n|LA_n}(LX_n^j|LA_n^j)f_{LA_n}(LA_n^j)\right\}\right]}{f_{LX_n,LA_n}(LX_n^j, LA_n^j)} \tag{28}$$

Thus, an estimator $\frac{\partial \widehat{F}^{-1}_{LX_n|LA_n}(u_n^j|LA_n^j)}{\partial LA_n}$ is obtained by replacing the unknown quantities by their estimates, namely

$$\widehat{F}_{LX_n|LA_n}(LX_n^j|LA_n^j) = \frac{\frac{1}{Jh_1 s_{LA_n}}\sum_{j'=1}^{J} 1(LX_n^{j'} \le LX_n^j)K\left(\frac{LA_n^j - LA_n^{j'}}{h_1 s_{LA_n}}\right)}{\frac{1}{Jh_1 s_{LA_n}}\sum_{j'=1}^{J} K\left(\frac{LA_n^j - LA_n^{j'}}{h_1 s_{LA_n}}\right)}$$

$$\frac{\partial \widehat{f}_{LA_n}(LA_n^j)}{\partial LA_n} = \frac{1}{Jh_1^2 s_{LA_n}^2}\sum_{j'=1}^{J} K'\left(\frac{LA_n^j - LA_n^{j'}}{h_1 s_{LA_n}}\right)$$

$$\frac{\partial}{\partial LA_n}\left(\widehat{F}_{LX_n|LA_n}(LX_n^j|LA_n^j)\widehat{f}_{LA_n}(LA_n^j)\right) = \frac{1}{Jh_1^2 s_{LA_n}^2}\sum_{j'=1}^{J} 1(LX_{ni}^{j'} \le LX_{ni}^j)K'\left(\frac{LA_n^j - LA_n^{j'}}{h_1 s_{LA_n}}\right)$$

$$\widehat{f}_{LX_n,LA_n}(LX_n^j, LA_n^j) = \frac{1}{Jh_2^2 s_{LX_n} s_{LA_n}}\sum_{j'=1}^{J} K\left(\frac{LX_{ni}^j - LX_{ni}^{j'}}{h_2 s_{LX_{ni}}}\right)K\left(\frac{LA_n^j - LA_n^{j'}}{h_2 s_{LA_n}}\right)$$

where s_{LX_n} and s_{LA_n} are the standard errors of the random variables LX_n and LA_n, respectively, h_1 and h_2 some bandwidth and $K(\cdot)$ some kernel with $K'(\cdot)$ its derivative.

Note that due to the discriminatory pricing one of the implications of our model is that for any given n and j, we have $\frac{X_n^j}{A_n^j} \leq P_n^{c,A,j}$, therefore whenever our estimate $\frac{\partial \widehat{F}^{-1}_{LX_n|LA_n}(u_n^j|LA_n^j)}{\partial LA_n} <$ 1, we consider this as a violation. Setting $h_1 = cJ^{-1/5}$ and $h_2 = cJ^{-1/6}$, we pick c such that those violations are minimized. In our case that c is 1.4039, close to the rule of thumb case and we have 167 violations out of 969 cases.[28] We use a standard Gaussian kernel in our estimation.

5 Nonparametric Estimation

After introducing the hypothetical good A and constructing the market clearing prices, our observations are $\left\{A_{ni}^j, P_n^{c,A,j}\right\}_{j=1}^J$ for each n and i. All the identification results in Section 3 are valid for the hypothetical good A case as well and we are after the structure $[F_{ni}(\cdot), c_{ni}^{\circ,A}(\cdot)]$. Estimation follows the same steps as identification: We first estimate $H_{ni}^A(\cdot,\cdot)$, then estimate $c_{ni}^{\circ,A}(\cdot)$ and lastly, we estimate $F_{ni}(\cdot)$.

5.1 Estimation of $H_{ni}^A(\cdot,\cdot)$

Analogous to (9) we have

$$H_{ni}^A(p_n, a_{ni}) = \int_0^\infty \left[\int_{a_n - a_{ni}}^\infty f_{\Sigma^A_{n,-i,p}}(a_{n,-i}) da_{n,-i} \right] f_{A_n}(a_n) da_n \tag{29}$$

[28] In this particular application, since we also have individual observations whenever our $P_n^{c,A,j}$ estimate is lower than $\max\limits_{i\neq n} \left\{ \frac{X_{ni}^j}{A_{ni}^j} \right\}$ we take that max value.

Moreover, from Lemma 1, applied to the hypothetical good A, we have

$$H^A_{ni}(p_n, a_{ni}) = 1 - \frac{\int_0^\infty \left\{ \int_0^\infty F_{A_{n,-i}|P^{c,A}_n, A_n}(a_n - a_{ni}|p_n, \widetilde{a}_n) dF_{A_n}(a_n) \frac{f_{P^{c,A}_n, A_n}(p_n, \widetilde{a}_n)}{f^2_{A_n}(\widetilde{a}_n)} \right\} dF_{A_n}(\widetilde{a}_n)}{\int_0^\infty \frac{f_{P^{c,A}_n, A_n}(p_n, \widetilde{a}_n)}{f^2_{A_n}(\widetilde{a}_n)} dF_{A_n}(\widetilde{a}_n)} \tag{30}$$

Expressing (30) in terms of expectations and our logarithmic transformations we get

$$H^A_{ni}(p_n, a_{ni}) = 1 - \frac{E_{\widetilde{A}_n}[\frac{f_{LP^{c,A}_n, LA_n}(\ln p_n, \ln \widetilde{A}_n)}{f^2_{LA_n}(\ln \widetilde{A}_n)} \widetilde{A}_n E_{A_n}[F_{LA_{n,-i}|LP^{c,A}_n, LA_n}(\ln(A_n - a_{ni})| \ln p_n, \ln \widetilde{A}_n)]]}{E_{\widetilde{A}_n}[\frac{f_{LP^{c,A}_n, LA_n}(\ln p_n, \ln \widetilde{A}_n)}{f^2_{LA_n}(\ln \widetilde{A}_n)} \widetilde{A}_n]} \tag{31}$$

Denote the numerator in equation (31) by $N_{ni}(p_n, a_{ni})$ and the denominator by $D_n(p_n)$. We propose the following estimator for $N_{ni}(p_n, a_{ni})$:

$$\widehat{N}_{ni}(p_n, a_{ni}) = \frac{1}{J} \sum_{j'=1}^{J} \left[\frac{A^{j'}_n}{\widehat{f}^2_{LA_n}(\ln A^{j'}_n)} \frac{1}{J} \sum_{j=1}^{J} \widehat{F}_{LA_{n,-i}|LP^{c,A}_n, LA_n}(\ln(A^j_n - a_{ni})| \ln p_n, \ln A^{j'}_n) \widehat{f}_{LP^{c,A}_n, LA_n}(\ln p_n, \ln A^{j'}_n) \right] \tag{32}$$

where

$$\begin{aligned} & \widehat{F}_{LA_{n,-i}|LP^{c,A}_n, LA_n}(\ln(A^j_n - a_{ni})| \ln p_n, \ln A^{j'}_n) \widehat{f}_{LP^{c,A}_n, LA_n}(\ln p_n, \ln A^{j'}_n) \\ = \; & \frac{1}{J h^2_3 s_{LP^{c,A}_n} s_{LA_n}} \sum_{k=1}^{J} 1(\ln(A^k_{n,-i}) \leq \ln(A^j_n - a_{ni}) K \left(\frac{\ln p_n - \ln(P^{c,A,k}_n)}{h_3 s_{LP^{c,A}_n}} \right) K \left(\frac{\ln A^{j'}_n - \ln A^k_n}{h_3 s_{LA_n}} \right) \end{aligned} \tag{33}$$

and

$$\widehat{f}^2_{LA_n}(\ln A^{j'}_n) = \frac{1}{J h^2_4 s^2_{LA_n}} \left[\sum_{k=1}^{J} K \left(\frac{\ln A^{j'}_n - \ln A^k_n}{h_4 s_{LA_n}} \right) \right]^2 \tag{34}$$

Similarly, we propose the following estimator for $D_n(p_n)$

$$\widehat{D}_n(p_n) = \frac{1}{J} \sum_{j'=1}^{J} \frac{\widehat{f}_{LP^{c,A}_n, LA_n}(\ln p_n, \ln A^{j'}_n)}{\widehat{f}^2_{LA_n}(\ln A^{j'}_n)} A^{j'}_n \tag{35}$$

where

$$\widehat{f}_{LP_n^{c,A},LA_n}(\ln p_n, \ln A_n^{j'}) = \frac{1}{Jh_3^2 s_{LP_n^{c,A}} s_{LA_n}} \sum_{k=1}^{J} K\left(\frac{\ln p_n - \ln(P_n^{c,A,k})}{h_3 s_{LP_n^{c,A}}}\right) K\left(\frac{\ln A_n^{j'} - \ln A_n^k}{h_3 s_{LA_n}}\right) \tag{36}$$

where h_3 and h_4 are some bandwidth and $K(\cdot)$ is some kernel.

For an estimate of $H_{ni,p}^A(\cdot,\cdot)$, we use the fact that

$$H_{ni,p}^A(p_n, a_{ni}) = \frac{\left[N_{ni}(p_n, a_{ni})\frac{\partial D_n(p_n)}{\partial p} - D_n(p_n)\frac{\partial N_{ni}(p_n,a_{ni})}{\partial p}\right]}{[D_n(p_n)]^2} \tag{37}$$

and replacing the numerator and denominator with their respective estimates proposed above, then, taking the derivative provides our estimator.

5.2 Estimation of $c_{ni}^{\circ,A}(\cdot)$

Estimation of $c_{ni}^{\circ,A}(\cdot)$ will be in two steps. First, we need to estimate the lower boundary of the support $\mathcal{S}_{A_{ni},P_n^c}$. We, then, estimate the cost frontier. In order to estimate the lower boundary $s_{ni}^{A-1}(\cdot,\bar{z}) = s_{ni}^{A-1}(\cdot,1)$ of the support of (A_{ni}, P_n^c), we use splines. The supply schedules are weakly increasing in a_{ni}, the quantity of good A supplied by player i in any destination n. Using splines we can guarantee that our estimator is continuous and smooth but we also like to impose the (weak) monotonicity condition. Following DeVore(1977), we integrate lower degree splines which provides weakly increasing functions by construction.

For any $t \in [0,1]$, we define

$$\widetilde{LB}(t) = \sum_{k=0}^{J_n+2} \psi_k B_k(t)$$

where $B_o(t) = 1$, $B_k(t) = \int_0^t b_k(u)du$ for $k = 1, ...J_n + 2$ and $b_k(\cdot)$'s are linear B-spline basis functions while J_n is the number of knots $\{t_l : t_o = 0 < t_1 < ... < t_{J_n+1} = 1, l = 0, 1, .., J_n+1\}$

in between the endpoints. Specifically, the $b_k(\cdot)$'s are as follows:

$$\begin{aligned}
\text{For } k &= 1, & b_1(u) &= \left[1-\frac{u}{t_1}\right]1(u\in[t_0,t_1]) \\
\text{For } k &= 2,...,J_n+1, & b_k(u) &= \left[\frac{u-t_{k-2}}{t_{k-1}-t_{k-2}}\right]1(u\in[t_{k-2},t_{k-1})) + \left[\frac{t_k-u}{t_k-t_{k-1}}\right]1(u\in[t_{k-1},t_k)) \\
\text{For } k &= J_n+2, & b_{J_n+2}(u) &= \left[\frac{u-t_{J_n}}{t_{J_n+1}-t_{J_n}}\right]1(u\in[t_{J_n},t_{J_n+1}])
\end{aligned}$$

We use "uniform B-splines" meaning that the knots are equally spaced, i.e. $t_{l-1}-t_{l-2} = t_l - t_{l-1} = \frac{1}{J_n+1}$ for any $l = 1,...,J_n+1$. We, then, have the following basis functions $B_k(t)$'s for all $t \in [0,1]$:

$$\text{For } k=0, \quad B_0(t)=1$$

$$\text{For } k=1, \quad B_1(t) = \left[t - \tfrac{t^2(J_n+1)}{2}\right]1(t\in[0,\tfrac{1}{J_n+1})) + \tfrac{1}{2(J_n+1)}1(\ t\in[\tfrac{1}{J_n+1},1])$$

$$\text{For } k=2,...,J_n+1,\ B_k(t) = \left[\frac{(t-\frac{k-2}{J_n+1})^2(J_n+1)}{2}\right]1(t\in[\tfrac{k-2}{J_n+1},\tfrac{k-1}{J_n+1}))$$

$$+\left[\frac{1}{(J_n+1)} - \frac{\left(\frac{k}{J_n+1}-t\right)^2(J_n+1)}{2}\right]1(t\in[\tfrac{k-1}{J_n+1},\tfrac{k}{J_n+1})) + \left[\frac{1}{J_n+1}\ \right]\ 1(t\in\tfrac{k}{J_n+1},1)$$

$$\text{For } k=J_n+2, \quad B_{J_n+2}(t) = \left[\frac{(t-\frac{J_n}{J_n+1})^2(J_n+1)}{2}\right]1(\ t\in[\tfrac{J_n}{J_n+1},1])$$

In order for $\widetilde{LB}(t)$ to be weakly increasing over $t\in[0,1]$, it is necessary and sufficient that $\psi_k \geq 0$ for $k = 1,...,J_n+2$.

We define $\widetilde{LA^j_{ni}} = \frac{LA^j_{ni} - \min_j(LA^j_{ni})}{\max_j(LA^j_{ni}) - \min_j(LA^j_{ni})}$ and $LB(A^j_{ni}) = \widetilde{LB}(\widetilde{LA^j_{ni}})$. Note that $\widetilde{LA^j_{ni}} \in [0,1]$ so we can substitute $\widetilde{LA^j_{ni}}$ for t in the above equations. Since $\widetilde{LB}(\cdot)$ is the lower boundary, our estimator is the curve that maximizes the area under it while having all the observations above, following Tsybakov (1993). In Campo, Guerre, Perrigne and Vuong (2009), they also follow a similar approach to estimate the upper boundary of the bid distribution. Specifically, we solve the following problem:

$$\max_{\{\psi_k\}_{k=0}^{J_n+2}} \int_0^1 \widetilde{LB}(\widetilde{LA}_{ni})d\widetilde{LA}_{ni}$$

$$\begin{aligned} s.t.\ P_n^{c,A,j} &\geq \widetilde{LB}(\widetilde{LA}_{ni}^j) \text{ for all } j = 1, ..., J \\ \psi_k &\geq 0 \text{ for all } k = 1, ..., J_n + 2 \end{aligned}$$

The first constraint imposes that all the observations lie above our $\widetilde{LB}(\cdot)$ curve and the second one imposes (weak) monotonicity. Substituting the definition of $\widetilde{LB}(\cdot)$ into our objective function and the constraint, the problem becomes a simple linear programming problem since both the objective function and the constraints are linear in ψ_k's.

Once we have $\widehat{s}_{ni}^{A\,-1}(\cdot, 1)$, our estimate of $c_{ni}^{\circ,A}(\cdot)$ is given by

$$\widehat{c}_{ni}^{\circ,A}(\cdot) = \widehat{s}_{ni}^{A\,-1}(\cdot, 1) - \frac{1 - \widehat{H}_{ni}^A(\widehat{s}_{ni}^{A\,-1}(\cdot, 1), \cdot)}{\widehat{H}_{ni,p}^A(\widehat{s}_{ni}^{A\,-1}(\cdot, 1), \cdot)} \tag{38}$$

5.3 Estimation of $F_{ni}(\cdot)$

Once we estimate the marginal cost frontier by $\widehat{c}_{ni}^{\circ,A}(\cdot)$, we can estimate the *pseudo-productivities* for any $(A_{ni}^j, P_n^{c,A,j})$

$$\widehat{z}_{ni}^j = \frac{\widehat{c}_{ni}^{\circ,A}(A_{ni}^j)}{P_n^{c,A,j} - \frac{1-\widehat{H}_{ni}^A(P_n^{c,A,j}, A_{ni}^j)}{\widehat{H}_{ni,p}^A(P_n^{c,A,j}, A_{ni}^j)}} \tag{39}$$

We, then, estimate the density $f_{ni}(\cdot)$ of productivities by

$$\widehat{f}_{ni}(z_{ni}) = \frac{1}{Jh_5 s_{\widehat{z}_{ni}}} \sum_{j=1}^{J} K\left(\frac{z_{ni} - \widehat{z}_{ni}^j}{h_5 s_{\widehat{z}_{ni}}}\right) \tag{40}$$

where $s_{\widehat{z}_{ni}}$ is the standard error of $\widehat{z}_{ni}$, h_5 is some bandwidth and $K(\cdot)$ is some kernel.

6 Empirical Results

As we mentioned before, for this particular application, we look at the German market for imports. Figure 2 shows our estimates of the base marginal cost $\widehat{c}^{\circ,A}_{ni}(\cdot)$ for Belgium and US and the associated markup when the exporters have the highest possible productivity. These figures are obtained based on our estimates (38) and $\widehat{s}^{A}_{ni}{}^{-1}(\cdot, 1)$. For the case of Germany, we know from Table 1 that Belgium is the third largest exporter in terms of export shares and US is the sixth. Our estimates reflect this fact that for this particular destination base marginal cost of Belgium is lower than US. As mentioned before, base marginal cost is the cost frontier of these countries, i.e. they assumed to have the highest level of productivity, $\overline{z} = 1$. Thus, the difference in base marginal costs can solely be attributed to trade barriers or input costs. Given the scale used in Figure 2, it might be hard to notice immediately, but for Belgium for smaller units marginal cost is not increasing that fast and it starts to get steeper for larger units, whereas for US this behavior is kind of reversed. For Belgium, the markup seems to be slightly declining , whereas for US, it is increasing. On the overall, however, US markups are higher than Belgium ones, even though they both have the highest productivity.

In Figure 3, we display the markup to marginal cost ratio. The reason we are interested in this ratio is that what we call markup and the standard markup in the monopolistic competition models are slightly different. There, markup is price over marginal cost which is constant across units whereas in our case what we call markup is price minus marginal cost, which may or may not be constant across units. In those models markup is not only constant over units, but also constant across exporters and even across markets. Therefore, in order to see for instance whether the markup is indeed constant say across exporters, we need to look at whether markup to marginal cost ratio is constant across countries or not. In Figure 3, we see that markups are not fixed across these two exporters.

Other than markups, another question which has become popular recently, is about the characteristics of the productivity distributions assumed in trade models and to what extent they effect the welfare gains. In Eaton and Kortum (2002) and many variants as mentioned earlier the distributional assumption is Fréchet. It has a key parameter θ that is used to measure the comparative advantage. Its estimation is important for calculating welfare gains. Simonovska and Waugh (2010) claim that their estimate of θ is actually half of the conventional estimates of θ, which causes a doubling in estimation of the welfare costs of autarky in those models. Therefore, it is important to see how the distribution of productivities behaves.

In Figure 4, we display the estimated denstiy $\widehat{f}_{ni}(\cdot)$ of estimated productivities for Belgium and in Figure 5 for US. Our first observation is that these distributions are not unimodal. It is almost a common assumption in the models we mentioned in trade that for any good countries draw from the same distribution. Our result, however, suggests that certain industries have certain types (levels) of productivity. This is important from a policy perspective since policy makers might want to distinguish between less productive and more productive types. Second, we observe that compared to Belgium there is more dispersion in the productivity distribution of US. Belgium's distribution is more concentrated around lower productivity levels.

Now, the big question : Is it really Fréchet ? According to our maximum likelihood estimates, we find $\widehat{\theta}^{*}_{Bel} = 0.4036$ for Belgium whereas the literature estimates of θ range from 3 to 13. As it can be seen from Figure 6, the truncated Frechet distribution with $\widehat{\theta}^{*}_{Bel} = 0.4036$ does not fit very well our nonparametric estimate. We also tried truncated Pareto case even though our analysis was at the country level, but the fit was too poor. Figure 7, shows the same two distributions for US. For US, our maximum likelihood estimates give $\widehat{\theta}^{*}_{US} = 0.2864$ and again the assocaiated truncated Fréchet does not fit well our nonparametric estimate.

7 Conclusion

In this paper, we introduce supply function competition to explain bilateral trade data. Unlike the convention in the existing literature in international trade, we keep a flexible structure for the productivity distributions and marginal costs. Next, we show that the productivity distributions and marginal costs can be identified nonparametrically. Due to the nature of the bilateral trade data only trade values and traded quantities can be observed, however, we show that the structure of our model can still be identified when only transaction points are observed instead of the whole schedule. Following the nonparametric identification we propose a nonparametric estimation procedure.

The 1990 German market for manufacturing imports was analyzed. Our empirical results show that markups are not constant across exporters, in contrast to monopolistic competition with Dixit-Stiglitz preferences, which implies a constant markup across exporters. In the recent literature there has been some attempts to introduce variable markups. Our model contributes to those attempts as well. Our results indicate that the productivity distributions are not unimodal and do not support the conventional distributional assumptions. Another interesting application would be to look at different markets. Again, we would like to know whether markups are constant across markets or whether in some markets markups are consistently high.

We consider the following possible extensions for future research : First, introducing zeros, i.e., cases when countries do not trade with each other. Our theoretical model allows for the fact that there can be zeros. Our identification argument, however, needs all countries in our sample to export a positive amount. Second, we would like to have a more general form of demand which allows for quantity demanded for imports to depend on the clearing price. In that case, as long as we can find home production data at a disaggragated level that matches our bilateral trade data we can include also the home country in our analysis. Third, in our identification argument it was easier to do the analysis using market clearing prices and quantities. That is why we used Pehlivan and Vuong (2010b) to construct market clearing prices. It might still be possible to obtain identification using only expenditures and quantities which is an open issue left for future research.

APPENDIX A

Derivation of the necessary condition

Similar to Hortacsu (2002), we will use calculus of variations to solve our functional optimization problem. For notational simplicity we will drop hereafter index j and n and the conditioning on its characteristics b^j, as the analysis is performed for a given good j and n.

Our expected profit maximization problem is defined as

$$\max_{s_i(\cdot,z_i)} \int_{\underline{p}}^{\overline{p}} \underbrace{\left\{ \int_0^{s_i(p^c,z_i)} \left[s_i^{-1}(q,z_i) - c_i(q,z_i) \right] \; dq \right\}}_{\pi(s_i(p^c,z_i))} \underbrace{d\widetilde{H}_i(p^c|z_i)}_{dH_i(p^c,s_i(p^c,z_i))}$$

Let $s_i(p^c, z_i)$ be defined over the support $[\underline{p}, \overline{p}]$

$$s_i(p^c,z_i) = \begin{cases} 0 & \text{if } \; \underline{p} \leq p^c \leq p_i^* \\ \widetilde{s}_i(p^c,z_i) & \text{if } \; p_i^* \leq p^c \leq \overline{p} \end{cases}$$

Let $I = \int_{\underline{p}}^{\overline{p}} \pi(s_i(p^c,z_i)) \; dH_i(p^c, s_i(p^c,z_i)) = \int_{p_i^*}^{\overline{p}} \pi(\widetilde{s}_i(p^c,z_i)) \; dH_i(p^c, \widetilde{s}_i(p^c,z_i))$ and note that by Leibnitz's Rule

$$\begin{aligned} \frac{d\pi}{dp^c} &= \left[\widetilde{s}_i^{-1}(\widetilde{s}_i(p^c,z_i),z_i) - c_i(q,z_i) \right] \widetilde{s}_i'(p^c,z_i) \\ &= \left[p^c - c_i(\widetilde{s}_i(p^c,z_i),z_i) \right] \widetilde{s}_i'(p^c,z_i) \end{aligned}$$

where $\widetilde{s}_i'(p^c, z_i)$ is the derivative of $\widetilde{s}_i(p^c, z_i)$ with respect to the first argument. Applying integration by parts and noting that $\pi(\widetilde{s}_i(p_i^*, z_i)) = 0$, we get

$$I = \pi(\widetilde{s}_i(\overline{p},z_i)) - \int_{p_i^*}^{\overline{p}} H_i(p^c, \widetilde{s}_i(p^c,z_i)) \left[p^c - c_i(\widetilde{s}_i(p^c,z_i),z_i) \right] \widetilde{s}_i'(p^c,z_i) dp^c$$

Note that $\pi(\widetilde{s}_i(\overline{p}, z_i)) = \int\limits_0^{\widetilde{s}_i(\overline{p}, z_i)} \left[\widetilde{s}_i^{-1}(q, z_i) - c_i(q, z_i)\right] \, dq$. Applying change of variable and setting $p_i^* = c_i(0, z_i)$, we have $\pi(\widetilde{s}_i(\overline{p}, z_i)) = \int\limits_{p_i^*}^{\overline{p}} [p^c - c_i(\widetilde{s}_i(p^c, z_i), z_i)] \, \widetilde{s}_i'(p^c, z_i) dp^c$. Then, our maximization problem becomes

$$\max_{\widetilde{s}_i(\cdot, z_i)} \int\limits_{p_i^*}^{\overline{p}} \left(1 - H_i(p^c, \widetilde{s}_i(p^c, z_i))\right) [p^c - c_i(\widetilde{s}_i(p^c, z_i), z_i)] \, \widetilde{s}_i'(p^c, z_i) dp^c$$

Since the integrand is a function of $p^c, \widetilde{s}_i$ and $\widetilde{s}_i'$ denote the integrand by $F_i(p^c, \widetilde{s}_i, \widetilde{s}_i')$. From Kamien and Schwartz (1993), the Euler equation which provides our necessary condition is given by

$$F_{i,\widetilde{s}_i} = \frac{dF_{i,\widetilde{s}_i'}}{dp^c}$$

where $F_{i,\widetilde{s}_i}$ is the partial derivative of $F_i(p^c, \widetilde{s}_i, \widetilde{s}_i')$ with respect to the second variable and $F_{i,\widetilde{s}_i'}$ is the partial derivative with respect to the third variable. Plugging in the respective derivatives we get

$$-H_{i,p}(p^c, \widetilde{s}_i(p^c, z_i)) \, [p^c - c_i(\widetilde{s}_i(p^c, z_i), z_i)] + (1 - H_i(p^c, \widetilde{s}_i(p^c, z_i))) = 0$$

for $p_i^* \leq p^c \leq \overline{p}$. Rearranging it gives

$$p^c - c_i(\widetilde{s}_i(p^c, z_i), z_i) = \frac{(1 - H_i(p^c, \widetilde{s}_i(p^c, z_i)))}{H_{i,p}(p^c, \widetilde{s}_i(p^c, z_i))}$$

for $p_i^* \leq p^c \leq \overline{p}$.

APPENDIX B

Proof of Lemma 1

We will prove Lemma 1 by proving equations (10), (12) and (15).

Proof of (10) :

Since $\Pr\left[Q_n \leq q_n | P_n^c = p_n\right] = \Pr\left[\Sigma_{n,p_n} \leq q_n | XD_{n,p_n} = 0\right]$, we know that

$$f_{Q_n|P_n^c}\left(q_n|p_n\right) = f_{\Sigma_{n,p_n}|XD_{n,p_n}}\left(q_n|0\right) = \frac{f_{\Sigma_{n,p_n},XD_{n,p_n}}\left(q_n,0\right)}{f_{XD_{n,p_n}}(0)} \tag{41}$$

Now, consider the following transformation

$$\begin{pmatrix} \Sigma_{n,p_n} \\ Q_n \end{pmatrix} \longrightarrow \begin{pmatrix} \Sigma_{n,p_n} \\ XD_{n,p_n} \end{pmatrix}$$

since the Jacobian is 1 we get

$$f_{\Sigma_{n,p_n},XD_{n,p_n}}\left(\widetilde{q}_n, x_n\right) = f_{\Sigma_{n,p_n},Q_n}\left(\widetilde{q}_n, x_n + \widetilde{q}_n\right)$$

evaluating at $\widetilde{q}_n = q_n$ and $x_n = 0$ gives $f_{\Sigma_{n,p_n},XD_{n,p_n}}\left(q_n, 0\right) = f_{\Sigma_{n,p_n},Q_n}\left(q_n, q_n\right)$. Plugging this back to (41) we obtain

$$f_{Q_n|P_n^c}\left(q_n|p_n\right) = \frac{f_{\Sigma_{n,p_n},XD_{n,p_n}}\left(q_n,0\right)}{f_{XD_{n,p_n}}(0)} = \frac{f_{\Sigma_{n,p_n},Q_n}\left(q_n,q_n\right)}{f_{XD_{n,p_n}}(0)} = \frac{f_{\Sigma_{n,p_n}}(q_n)f_{Q_n}(q_n)}{f_{XD_{n,p_n}}(0)} \tag{42}$$

where the last equality follows from independence of Σ_{n,p_n} and Q_n since for any arbitrary p_n they are independent by Assumption A3. Rewriting (42) we have

$$f_{Q_n|P_n^c}\left(q_n|p_n\right) f_{XD_{n,p}}(0) = f_{\Sigma_{n,p_n}}(q_n)f_{Q_n}(q_n) \tag{43}$$

for all $(q_n, p_n) \in \mathbb{R}_+ \times \mathcal{S}_{P_n^c}$. Let $C_{p_n} = f_{XD_{n,p}}(0)$ and apply Bayes' Rule in (43), then we get

$$\frac{f_{P_n^c|Q_n}(p_n|q_n)\, C_{p_n}}{f_{P_n^c}(p_n)} = f_{\Sigma_{n,p_n}}(q_n) \tag{44}$$

for all $(q_n, p_n) \in \mathbb{R}_+ \times \mathcal{S}_{P_n^c}$. To get the C_{p_n} integrate both sides over q_n, the RHS will be 1, thus we get

$$C_{p_n} = \frac{f_{P_n^c}(p_n)}{\int\limits_{\mathcal{S}_{\Sigma_{n,p_n}}} f_{P_n^c|Q_n}(p_n|\widetilde{q}_n)\, d\widetilde{q}_n}$$

By Assumption A3 we have $\mathcal{S}_{\Sigma_{n,p_n}} = \mathcal{S}_{Q_n} = [0, \infty)$, hence

$$C_{p_n} = \frac{f_{P_n^c}(p_n)}{\int\limits_0^\infty f_{P_n^c|Q_n}(p_n|\widetilde{q}_n)\, d\widetilde{q}_n}$$

for any $p_n \in \mathcal{S}_{P_n^c}$. Substituting C_{p_n} into (44) yields

$$f_{\Sigma_{n,p_n}}(q_n) = \frac{f_{P_n^c|Q_n}(p_n|q_n)}{\int\limits_0^\infty f_{P_n^c|Q_n}(p_n|\widetilde{q}_n)\, d\widetilde{q}_n} \tag{45}$$

which is equation (10). $\square$

Proof of Equation (12) :

Since $\Pr[Q_{n,-i} \leq q_{n,-i}|P_n^c = p_n,\ Q_n = q_n] = \Pr[\Sigma_{n,-i,p_n} \leq q_{n,-i}|\Sigma_{n,p_n} = q_n, Q_n = q_n]$, it implies

$$f_{Q_{n,-i}|P_n^c,Q_n}(q_{n,-i}|p_n, q_n) = f_{\Sigma_{n,-i,p_n}|\Sigma_{n,p_n},Q_n}(q_{n,-i}|q_n, q_n) = \frac{f_{\Sigma_{n,-i,p_n},\Sigma_{n,p_n},Q_n}(q_{n,-i}, q_n, q_n)}{f_{\Sigma_{n,p_n},Q_n}(q_n, q_n)} \tag{46}$$

Now, consider the following transformation

$$\begin{pmatrix} \Sigma_{n,-i,p_n} \\ S_{ni,p_n} \\ Q_n \end{pmatrix} \longrightarrow \begin{pmatrix} \Sigma_{n,-i,p_n} \\ \Sigma_{n,p_n} \\ Q_n \end{pmatrix}$$

Since the Jacobian is 1 again, we have

$$f_{\Sigma_{n,-i,p_n},\Sigma_{n,p_n},Q_n}(\widetilde{q}_{n,-i},\widetilde{q}_n,\widetilde{\widetilde{q}}_n) = f_{\Sigma_{n,-i,p_n},S_{ni,p_n},Q_n}(\widetilde{q}_{n,-i},\widetilde{q}_n - \widetilde{q}_{n,-i},\widetilde{\widetilde{q}}_n)$$

evaluating at $\widetilde{q}_{n,-i} = q_{n,-i}$, $\widetilde{q}_n = q_n$ and $\widetilde{\widetilde{q}}_n = q_n$ we get

$$f_{\Sigma_{n,-i,p_n},\Sigma_{n,p_n},Q_n}(q_{n,-i},q_n,q_n) = f_{\Sigma_{n,-i,p_n},S_{ni,p_n},Q_n}(q_{n,-i},q_n - q_{n,-i},q_n)$$

When we plug this back into (46)

$$\begin{aligned} f_{Q_{n,-i}|P_n^c,Q_n}(q_{n,-i}|p_n,q_n) &= \frac{f_{\Sigma_{n,-i,p_n},\Sigma_{n,p_n},Q_n}(q_{n,-i},q_n,q_n)}{f_{\Sigma_{n,p_n},Q_n}(q_n,q_n)} \\ &= \frac{f_{\Sigma_{n,-i,p_n},S_{ni,p_n},Q_n}(q_{n,-i},q_n - q_{n,-i},q_n)}{f_{\Sigma_{n,p_n},Q_n}(q_n,q_n)} \\ &= \frac{f_{\Sigma_{n,-i,p_n}}(q_{n,-i}) f_{S_{ni,p_n}}(q_n - q_{n,-i})}{f_{\Sigma_{n,p_n}}(q_n)} \end{aligned} \tag{47}$$

where the last equality follows from independence of $\Sigma_{n,-i,p_n}$, S_{ni,p_n} and Q_n for any arbitrary p_n they are independent by Assumption A3. Note that we need Assumption A5 for the existence of $f_{Q_{n,-i}|P_n^c,Q_n}(q_{n,-i}|p_n,q_n)$. Rewriting (47) we get

$$f_{Q_{n,-i}|P_n^c,Q_n}(q_{n,-i}|p_n,q_n)\, f_{\Sigma_{n,p_n}}(q_n) = f_{\Sigma_{n,-i,p_n}}(q_{n,-i}) f_{S_{ni,p_n}}(q_n - q_{n,-i}) \tag{48}$$

for any $(q_{n,-i},p_n,q_n) \in \mathbb{R}_+ \times \mathcal{S}_{P_n^c} \times \mathcal{S}_{Q_n}$. Integrating both sides of (48) over q_n, remembering

$\mathcal{S}_{Q_n} = [0, \infty)$ yields

$$\int_0^\infty f_{Q_{n,-i}|P_n^c,Q_n}\left(q_{n,-i}|p_n,\widetilde{q}_n\right) f_{\Sigma_{n,p_n}}(\widetilde{q}_n)d\widetilde{q}_n = f_{\Sigma_{n,-i,p_n}}(q_{n,-i}) \int_0^\infty f_{S_{ni,p_n}}(\widetilde{q}_n - q_{n,-i})d\widetilde{q}_n$$

for any $(q_{n,-i}, p_n) \in \mathbb{R}_+ \times \mathcal{S}_{P_n^c}$. Consider the integral on the RHS, using change of variable it can be shown that it adds up to 1. Hence, we have

$$f_{\Sigma_{n,-i,p_n}}(q_{n,-i}) = \int_0^\infty f_{Q_{n,-i}|P_n^c,Q_n}\left(q_{n,-i}|p_n,\widetilde{q}_n\right) f_{\Sigma_{n,p_n}}(\widetilde{q}_n)d\widetilde{q}_n \tag{49}$$

for any $(q_{n,-i}, p_n) \in \mathbb{R}_+ \times \mathcal{S}_{P_n^c}$ which is equation (12). $\square$

Proof of Equation (15) :

From (45) plug the expression for $f_{\Sigma_{n,p_n}}(\cdot)$ into (49), then we get

$$f_{\Sigma_{n,-i,p_n}}(q_{n,-i}) = \int_0^\infty f_{Q_{n,-i}|P_n^c,Q_n}\left(q_{n,-i}|p_n,\widetilde{q}_n\right) \frac{f_{P_n^c|Q_n}\left(p_n|\widetilde{q}_n\right)}{\int_0^\infty f_{P_n^c|Q_n}\left(p_n|\widetilde{q}_n\right) d\widetilde{q}_n} d\widetilde{q}_n$$

rewriting yields

$$f_{\Sigma_{n,-i,p_n}}(q_{n,-i}) = \frac{\int_0^\infty f_{Q_{n,-i},P_n^c|Q_n}\left(q_{n,-i}|p_n,\widetilde{q}_n\right) d\widetilde{q}_n}{\int_0^\infty f_{P_n^c|Q_n}\left(p_n|\widetilde{q}_n\right) d\widetilde{q}_n}$$

for any $(q_{n,-i}, p_n) \in \mathbb{R}_+ \times \mathcal{S}_{P_n^c}$ which is equation (15). $\square$

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Table 1. Percentage of Manufacturing Imports for n = Germany	
France	16.6
Italy	14.0
Belgium	10.5
Japan	9.8
Netherlands	9.5
United States	9.4
United Kingdom	8.8
TOTAL	78.6

Note: Percentage of value of total manufacturing imports in the Eaton and Kortum (2002) sample.

Table 2. Coefficient of Variation of Unit Values across Exporters	
Mean	0.67
St. Deviation	0.40
Min	0.04
Max	2.56

Note: Coefficient of Variation is the ratio of st. deviation

of unit values to mean of unit values across exporters.

Table 3. Summary Statistics for A_{ni} for n =Germany (in dollars)

	Mean	St.Dev.	Min	Max
A_{Bel}	24,778,078	240,691,924	233	7,302,002,850
A_{Fra}	31,232,944	170,209,649	20	4,830,644,752
A_{Ita}	32,687,539	131,568,945	48	2,966,307,956
A_{Jpn}	24,383,660	323,489,959	2	9,892,275,446
A_{Ned}	21,252,720	51,390,764	93	680,094,647
A_{UK}	17,793,301	82,934,322	110	1,216,429,607
A_{US}	12,376,996	65,115,838	11	1,307,815,373

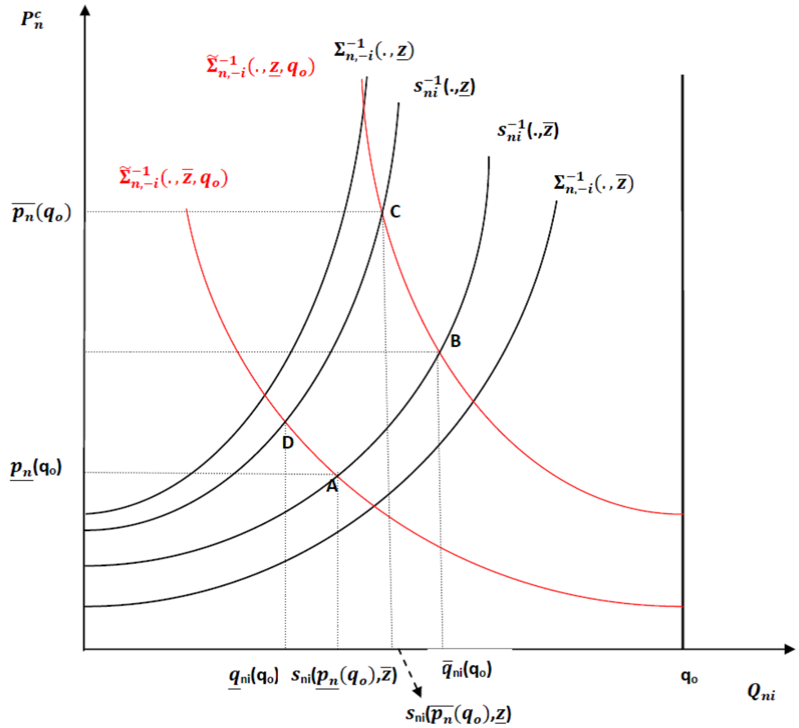


FIGURE 1: Identification of $c^{\circ}_{ni}(\cdot)$

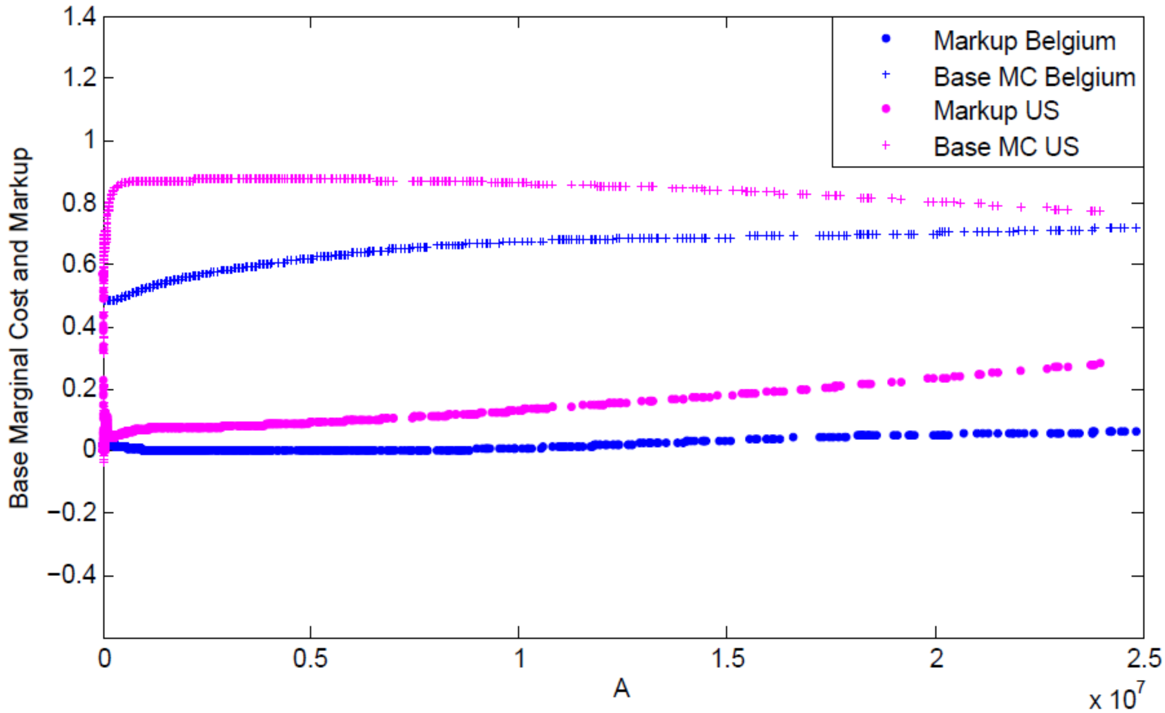


FIGURE 2: Estimates of $\widehat{c}_{ni}^{\infty,A}(\cdot)$ and Markup for Belgium and US in Germany

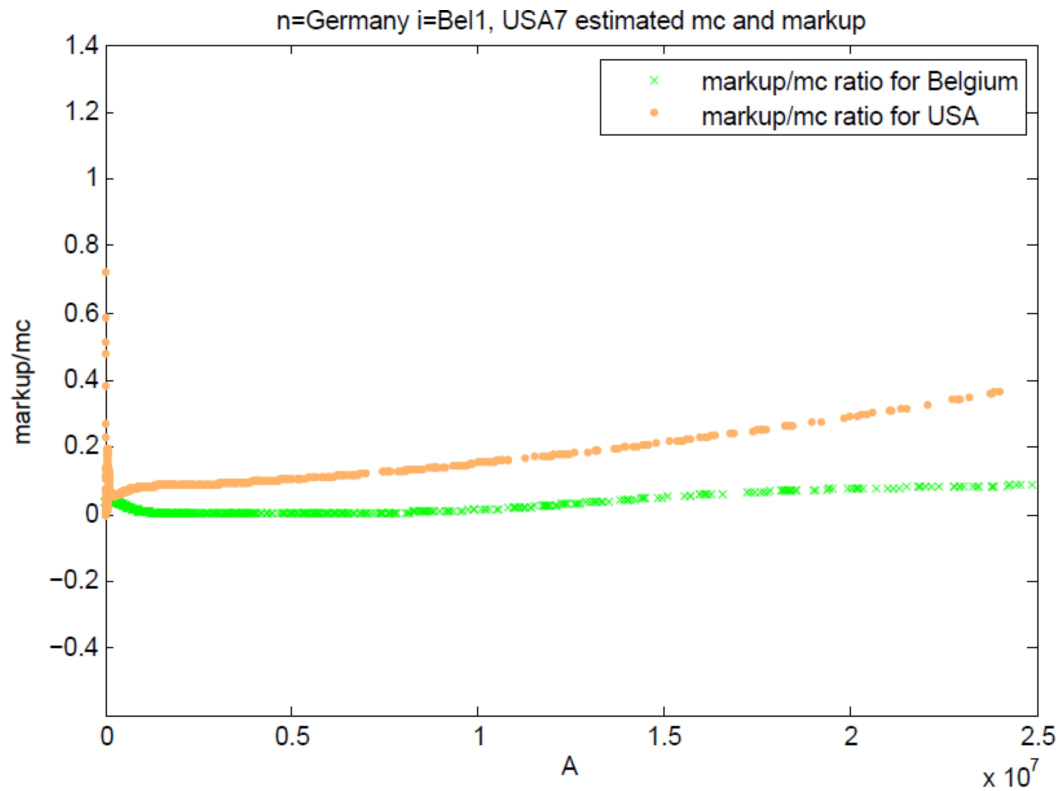

n=Germany i=Bel1, USA7 estimated mc and markup
markup/mc ratio for Belgium
markup/mc ratio for USA
markup/mc
A
x 10^7
1.4
1.2
1
0.8
0.6
0.4
0.2
0
−0.2
−0.4
0
0.5
1
1.5
2
2.5

FIGURE 3:

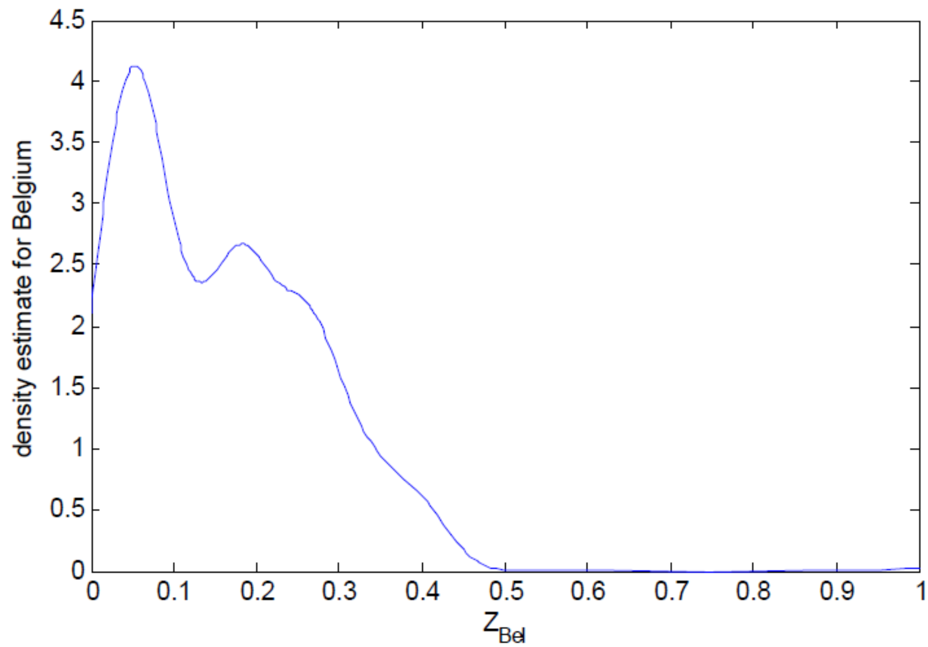


FIGURE 4: $\widehat{f}_{ni}(\cdot)$ for Belgium

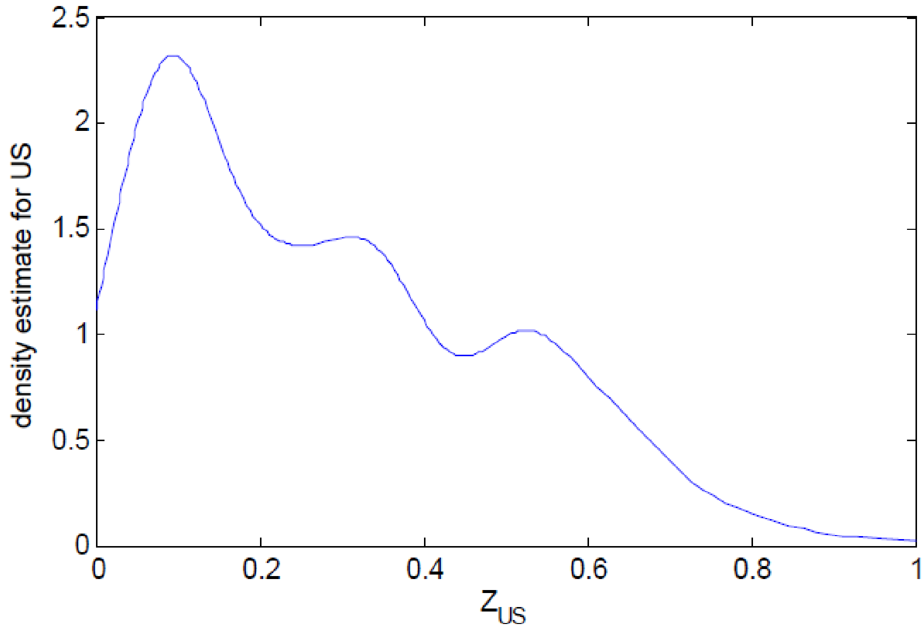
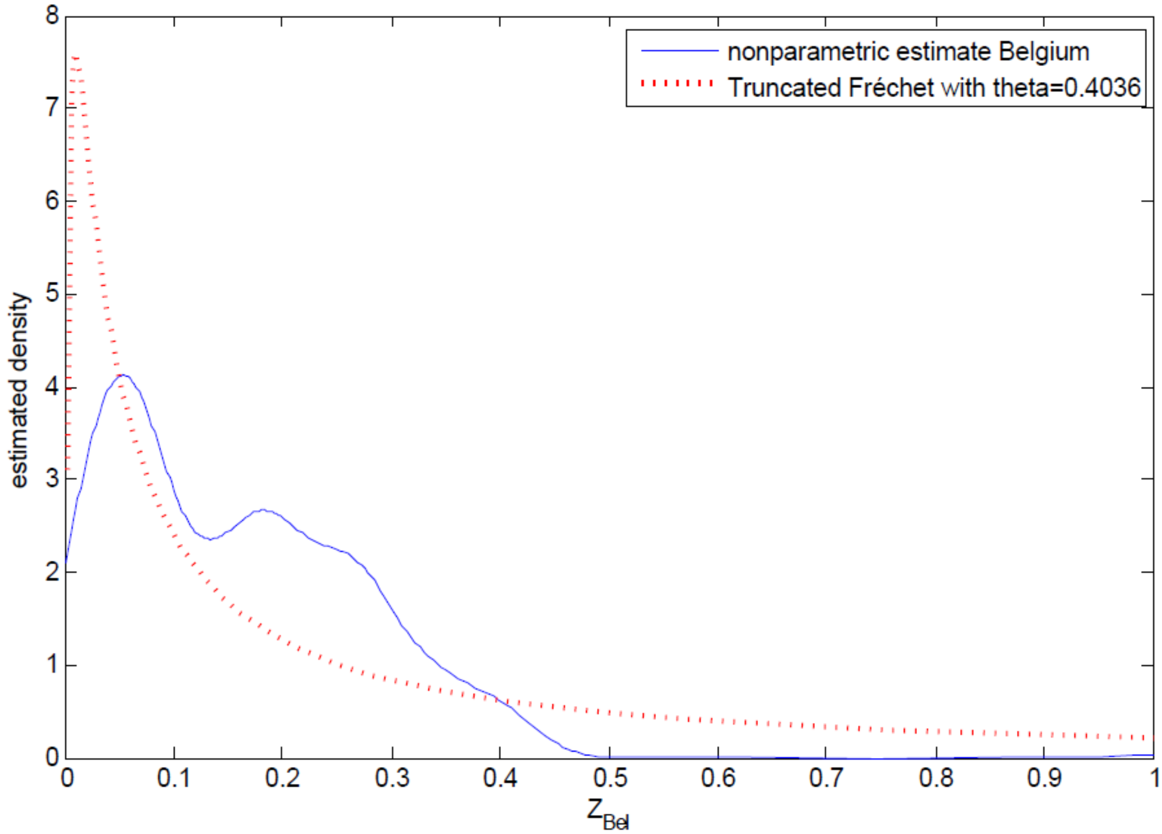


FIGURE 5: $\widehat{f}_{ni}(\cdot)$ for US



nonparametric estimate Belgium
Truncated Fréchet with theta=0.4036
estimated density
Z_Bel
0
1
2
3
4
5
6
7
8
0.1
0.2
0.3
0.4
0.5
0.6
0.7
0.8
0.9
1

FIGURE 6:

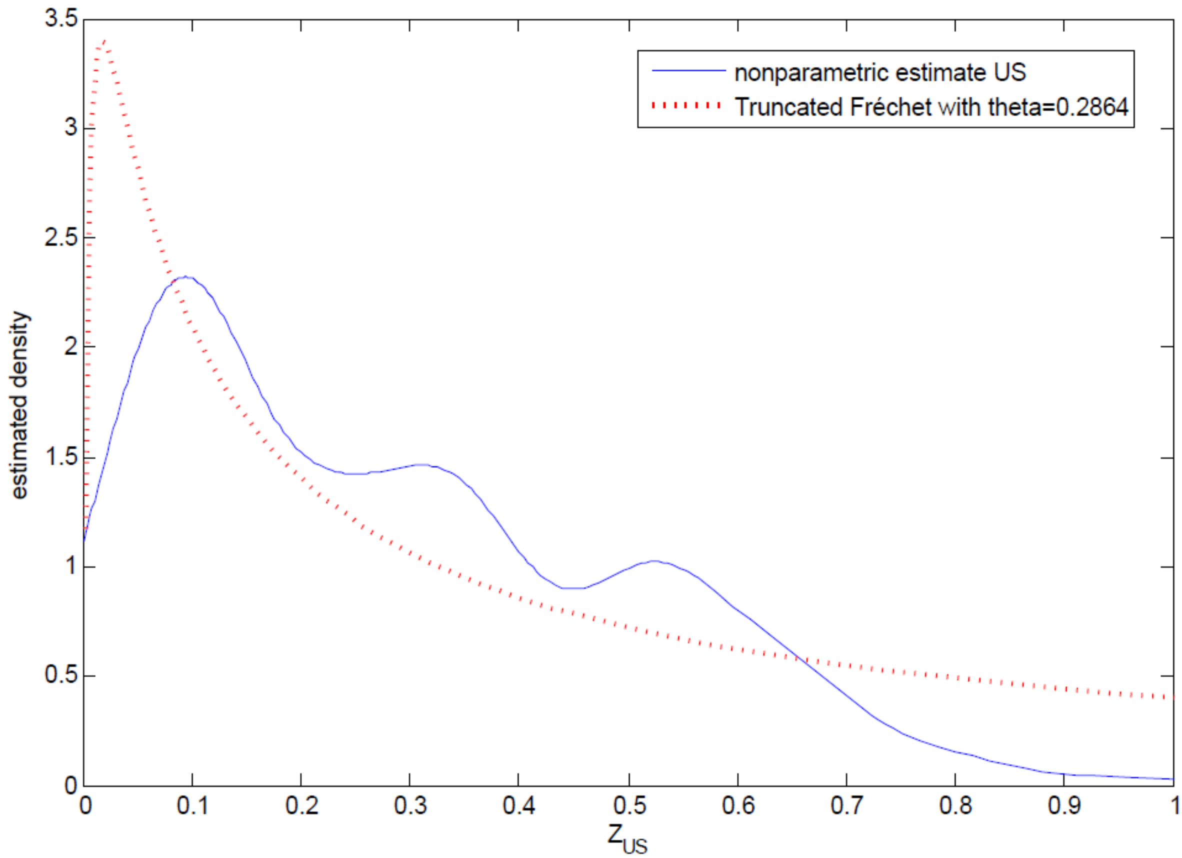

3.5
3
2.5
2
1.5
1
0.5
0
estimated density
nonparametric estimate US
Truncated Fréchet with theta=0.2864
0
0.1
0.2
0.3
0.4
0.5
0.6
0.7
0.8
0.9
1
Z_US

FIGURE 7: