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Spectral analysis of time-dependent market-adjusted return correlation matrix

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HIGHLIGHTS

- We present an adjusted method for the eigenvalues of a time-dependent correlation matrix.
- We use the correlation matrix on an 18-year data set of 310 S&P 500-listed stocks.
- We obtain more stationary eigenvalue time series via market-adjusted return.
- Co-movement and polarization of financial variables are important.
- We find an approximate polarization domain, characterized by a smooth L-shaped strip.

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ABSTRACT

We present an adjusted method for calculating the eigenvalues of a time-dependent return correlation matrix in a moving window. First, we compare the normalized maximum eigenvalue time series of the market-adjusted return correlation matrix to that of the logarithmic return correlation matrix on an 18-year dataset of 310 S&P 500-listed stocks for small and large window or memory sizes. We observe that the resulting new eigenvalue time series is more stationary than the time series obtained without the adjustment. Second, we perform this analysis while sweeping the window size $\tau \in \{5, \ldots, 100\} \cup \{500\}$ in order to examine the dependence on the choice of window size. This approach demonstrates the multi-modality of the eigenvalue distributions. We find that the three dimensional distribution of the eigenvalue time series for our market-adjusted return is significantly more stationary than that produced by classic method. Finally, our model offers an approximate polarization domain characterized by a smooth L-shaped strip. The polarization with large amplitude is revealed, while there is persistence in agreement of individual stock returns' movement with market with small amplitude most of the time.

1. Introduction

It is important to analyze the relative behavior of N particles with respect to changes in physical quantities. Empirical correlation matrices appear in similar complex problems of stock markets. The correlation between stock returns has been a fundamental component to the mathematics of finance since Markowitz first introduced the theory of portfolio selection [1,2]. Since then, time-dependent correlation within and across markets has been considered in the dynamics of

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assets on various time scales ranging from minutes to years [3–17]. Recently, the methods developed in applied spectral analysis and random matrix theory (RMT) have received significant attention in the study of correlation in financial markets. Many of these studies have focused on evaluating theoretical predictions [18–22] and uncovering interesting substructures and behaviors in markets [23–25]. Generally, they decompose the correlation matrix into market, group (sector), and the Wishart random bulk (noise terms) where the largest eigenvalue corresponds to market. [26] examines the daily returns of 422 US stocks for the time period 1962–96 and analyzes the deviating eigenvectors corresponding to the eigenvalues outside the random matrix theory bounds after removing the effect of the largest eigenvalue. [26] argues that the second largest eigenvalue of the empirical correlation matrix corresponds to large capitalization stocks. [14] finds that the deviating eigenvectors are stable in time. Recently, [27] finds long-range magnitude cross-correlations in time series of price fluctuations for 1340 members of the New York Stock Exchange (NYSE) Composite using time-lag random matrix theory. [28] discusses noise spectrum of the correlation matrix eigenvalues and the role of non-stationarity using Brazilian assets.

A common method of analysis in these studies is to observe the time series of the maximum eigenvalue of the time-dependent return correlation matrix. Recent literature has begun to investigate the use of this maximum eigenvalue time series in risk management and trading strategies [29]. [29] successfully incorporates a two-directional maximum eigenvalue of the time-dependent correlation matrix of the logarithmic returns into a moment-based percentile classification algorithm as a measure of risk. This signal prevents the strategy from trading in time periods where the eigenvalue of current window is relatively either too low (silence around overvalued price level before a storm in market) or too high (panic in market). Moreover, [29] provides compelling evidence for the usage of 100-session windows for correlation analysis in real market applications. Although the Marchenko–Pastur formula [30] is not applicable for this window size, the resulting signal is informative because of the long memory related to quarterly earnings reports. In other words, random matrix theory may not be applicable directly for some empirical correlation matrices due to its independence assumption and limitation on window size. Pairwise correlations between assets matter [31]. The resulting strategy in [29] outperforms the proxy for market in out-of-sample Monte Carlo simulations with random subsets of assets on random subperiods of the dataset.

Despite these significant contributions in literature, some issues with the usage of the maximum eigenvalue of empirical return correlation matrices remain. One such issue is that the time series corresponding to the maximum eigenvalue is highly non-stationary. It is important to obtain a more stationary time series in mathematical or physical modeling. One of our main goals in this paper is to find a relatively more stationary time series of maximum eigenvalue by using appropriate transformations. We have not preferred global nonlinear over-transformations or under-transformations such as logarithmic transformation. Instead, we focus on local transformation using relativity. We compare the resulting two dimensional and three dimensional time dependent maximum eigenvalue distributions. We obtain a relatively more stationary time series in mean value and standard deviation by using time series of market adjusted return (MAR).

The remainder of this paper is organized as follows. In Section 2, we introduce market-adjusted return and formalize our modified method. In Section 3, we describe the dataset constructed in order to test this method. In Section 4, we present the results of experiments on this dataset that evaluate the effect of our modification on stationarity. We compare kurtosis, autocorrelation function, and three dimensional maximum eigenvalue distribution values for both methods. Section 5 concludes the paper.

2. Market-Adjusted Return (MAR)

One of the fundamental assumptions in time-dependent correlation analysis is that the covariance structure of the underlying multivariate return process may not be constant or even stationary [32]. Most studies thus far have focused on modeling the covariance structure of this return process directly. In portfolio management, however, the return process of a portfolio can be sometimes misleading. Even if an individual asset has a positive and increasing rate of return over some window, this does not necessarily make it more attractive than the market. Moreover, just because two assets have positive correlation of return does not imply that their excess returns relative to market are positively correlated. A market-adjusted return (MAR) of an asset is simply the difference between the asset's return and a market index's return. Risk-adjusted returns typically consider the amount of MAR per unit risk. We focus on the correlation between the daily returns of assets. Since there is no natural way to measure the risk of an asset from a single data point in an out-of-sample algorithm, we cannot normalize market-adjusted return by risk, for example standard deviation. However, we can still better capture the movements and deviation behavior of assets relative to the market by examining the correlation between the market-adjusted daily returns of assets. Market-adjusted returns and risk-adjusted returns are identical in sign, and thus the parity of dyadic asset relationships is still preserved.

Before describing our modified method, we present an overview of the original method as presented in previous literature. The method takes as input an $M \times N$ return matrix R, where $R_{i,j}$ is the logarithmic return of the jth asset on the ith period, and τ is the window or memory size for calculating time-dependent correlation. For each time $t \in (\tau + 1, M)$, the windowed estimate of the mean $\hat{\mu}^{\tau}(t)$ and standard deviation $\hat{\sigma}^{\tau}(t)$ of each asset's return is calculated. We can explicitly write this windowed estimate of mean and standard deviation with window size τ at time t for the jth asset as follows:

$$\hat{\mu}_j^{\tau}(t) = \frac{1}{\tau} \sum_{i=t-\tau+1}^{t} R_{i,j} \tag{1}$$

$$\hat{\sigma}_j^{\tau}(t) = \sqrt{\frac{1}{\tau-1} \sum_{i=t-\tau+1}^{t} (R_{i,j} - \hat{\mu}_j^{\tau}(t))^2}. \tag{2}$$

These are the standard sample estimates of mean and standard deviation with fixed window often used in practice in risk management and trading. From these first two moments, we can then write the time-dependent empirical correlation matrix $C^{\tau}(t)$ with window size τ at time t as

$$C_{j,k}^{\tau}(t) = \frac{\sum_{i=t-\tau+1}^{t} (R_{i,j} - \hat{\mu}_j^{\tau}(t))(R_{i,k} - \hat{\mu}_k^{\tau}(t))}{(\tau-1)\hat{\sigma}_j^{\tau}(t)\hat{\sigma}_k^{\tau}(t)}. \tag{3}$$

Spectral methods then consider the set of eigenvalues λ_i and eigenvectors ν_i that satisfy the following equation

$$C^{\tau}(t)\vec{\nu_i} = \lambda_i \vec{\nu_i}. \tag{4}$$

To study risk-management and trading, [18] and [29] used a small number of the eigenvalues such as λ_{max}. As discussed above, [29] applies the normalized maximum eigenvalue $\Lambda = \frac{\lambda_{max}}{N}$ in a trading strategy with a window size of $\tau = 100$. A percentile trigger of Λ significantly improves the risk-adjusted return of their trading strategy in out-of-sample Monte Carlo backtesting. Other normalizations of eigenvalues such as those considered in [18] are also possible.

Our contribution is to modify this method to consider the time-dependent correlation of *market-adjusted* return. To do so, we need to subtract off the "market" rate of return from each asset's return for each period and then recalculate the first two moments. Eqs. (1)–(3) above can thus be rewritten as follows:

$$\hat{\mu}_j^{\tau}(t) = \frac{1}{\tau} \sum_{i=t-\tau+1}^{t} (R_{i,j} - m_i)$$

$$\hat{\sigma}_j^{\tau}(t) = \sqrt{\frac{1}{\tau-1} \sum_{i=t-\tau+1}^{t} (R_{i,j} - m_i - \hat{\mu}_j^{\tau}(t))^2}$$

$$C_{j,k}^{\tau}(t) = \frac{\sum_{i=t-\tau+1}^{t} (R_{i,j} - m_i - \hat{\mu}_j^{\tau}(t))(R_{i,k} - m_i - \hat{\mu}_k^{\tau}(t))}{(\tau-1)\hat{\sigma}_j^{\tau}(t)\hat{\sigma}_k^{\tau}(t)}$$

where m_i, the market return for period i, is approximated by $\frac{1}{N}\sum_j^N R_{i,j}$. Once this correlation matrix has been estimated, we again solve for the adjusted eigenvalues λ_i^* and eigenvectors ν_i^* implied by Eq. (4). We thus have $\Lambda^* = \frac{\lambda_{max}^*}{N}$. In Section 4 below, we will investigate the differences between Λ and Λ^* in more detail.

3. Dataset

In order to carry out the experiments in Section 4 below, we construct a dataset representing the current members of the S&P 500 over the period from 1991 to 2010. The adjusted price and volume data for each asset is obtained from Yahoo! Finance. Yahoo! Finance adjusts the price of assets after dividend and split events. This adjustment ensures that the return calculated from the adjusted prices reflects the true return derived from holding the asset after dividend and split events. Assets that were temporarily delisted for more than one trading session, had sessions with zero shares traded, or were not publicly traded as of January 1st, 1992 are also removed from the dataset. This additional selection filter is designed to increase the span and quality of the dataset. Furthermore, it ensures that the resulting assets are all comparable to each other over the time period of interest. The resulting dataset contains $N = 310$ assets and includes some of the oldest and largest publicly traded corporations in the United States.

Since there is some slight variation in the days on which these 310 assets have been traded over the past 18 years, we then determine the intersection of dates across all assets. That is, dates are discarded only unlike [29] where assets that were delisted even one trading session are removed from the dataset. The result is a set of 4508 dates out of the 4566 business days between 1991 and 2010. As an example, the NYSE was closed on the three business days immediately after September 11th, 2001.

We need to calculate the log-return matrix R before considering the correlation of assets. R can be calculated from the price matrix P simply by setting $R_{i+1,j} = \log(P_{i+1,j}) - \log(P_{i,j})$. R necessarily has one row fewer than P to reflect the fact that it is a derivative of observed data. m_i, the market return for period i, is given as above by $\frac{1}{N}\sum_j^N R_{i,j}$. Since the composition of the S&P 500 has changed considerably since 1992, it does not make sense to use the actual return of SPX. Instead, we consider the "market" return to be the return of the equal-weighted portfolio of assets that meet the selection criteria. Moreover, the S&P 500 index may overestimate the performance of the stock market. Fig. 1 shows the cumulative "market" log-return between 1991 and 2010.

Fig. 1. Cumulative market log-return, 1991–2010.

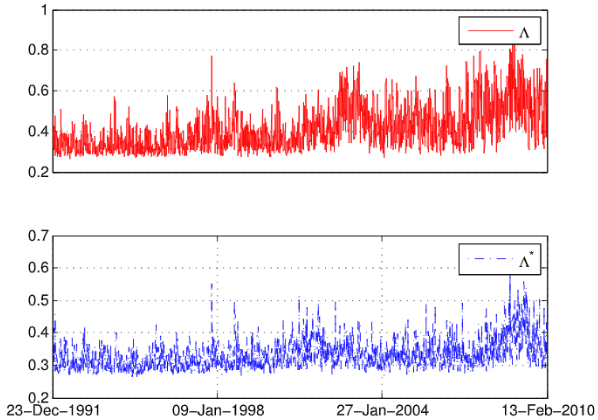

Fig. 2. Time series comparison of Λ and Λ^* for window size $\tau = 5$, 1991–2010.

4. Eigenvalue results

We would like to compare the original method and modified method presented in Section 2. In order to do this, we first use the dataset presented in Section 3 above to calculate the distribution of Λ and Λ^* for window sizes $\tau = 5$ and $\tau = 100$. Second, we calculate the distribution of Λ and Λ^* for all values of $\tau \in \{5, \ldots, 100\}$ and investigate the resulting surfaces to check if the findings are sensitive to τ.

4.1. 5-Day window ($\tau = 5$)

In order to approximate "instantaneous" correlation, we first consider a window size of $\tau = 5$. This window size corresponds to a 5 trading sessions or roughly the week-over-week period. The Marchenko–Pastur Q in this case is $\frac{\tau}{N} = \frac{5}{310} < 1$ and not applicable. The results of the original and modified calculations are presented below.

Fig. 2 shows the time series plots of maximum return correlation eigenvalue Λ and the maximum market-adjusted return correlation eigenvalue Λ^* over the period from 1991 to 2010. The scale on this figure clearly shows that the eigenvalue corresponding to market-adjusted return correlation is smaller than the return correlation eigenvalue over this period. To be precise, the market-adjusted return eigenvalue is less than the return eigenvalue for 84% of sessions. This supports the expectation that the analysis of market-adjusted return at least partially reduces the influence of the "market" and its corresponding eigenvector and eigenvalues. One financial interpretation can be that there is persistence in agreement of individual stock returns' movement with market most of time among the maximum proportion of variance explained by market. The remaining time may corresponds to relatively more polarization.

The nature of this difference is made more clear in Fig. 3, which shows the corresponding empirical probability densities for the eigenvalues from the original and modified methods. Though their peaks roughly co-occur, the tail of the market-adjusted return eigenvalue density decays significantly faster than that of the return eigenvalue's density. This represents

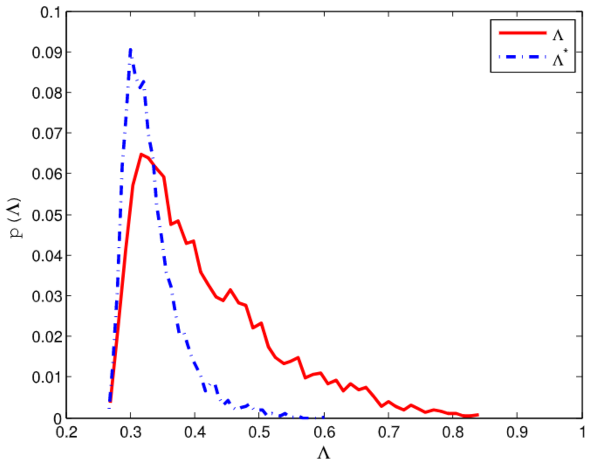

Fig. 3. Distribution of Λ and Λ^* for window size $\tau = 5$, 1991–2010.

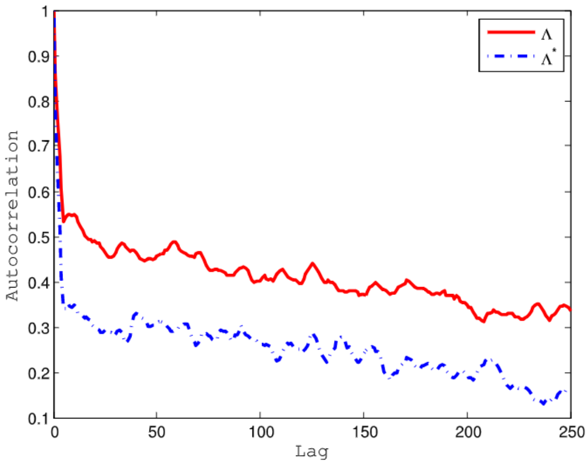

Fig. 4. Autocorrelation of Λ and Λ^* for window size $\tau = 5$.

the fact that the market-adjusted return eigenvalue exhibits much more peakedness — the kurtosis of the market-adjusted return eigenvalue distribution is 6.22, nearly twice the kurtosis value of 3.79 for the return distribution.

Fig. 4 shows the autocorrelation function (ACF) of Λ and Λ^*. Autocorrelation is a measure of how correlated a time series is with lagged versions of itself. More stationary time series will have ACFs with smaller magnitude values. In our case, we should especially expect the value of the ACF for lags past τ to be near 0. The market-adjusted return eigenvalue's ACF is strictly less than the ACF of the return eigenvalue. Furthermore, though both the market-adjusted return and return ACFs jump down at τ, the market-adjusted return decreases faster than the return ACF as the lag increases. This indicates that the market-adjusted eigenvalue time series is more stationary than the unadjusted return eigenvalue time series.

4.2. Large windows ($\tau = 100$ and $\tau = 500$)

Though the above results consider "instantaneous" correlation, most of the existing literature has considered much larger window sizes in their estimation of correlation. We repeat the analysis above after recalculating the output with the window size used in [29], $\tau = 100$.

Fig. 5 again shows the time series plot of maximum return correlation eigenvalue Λ and the maximum market-adjusted return correlation eigenvalue Λ^* over the period from 1991 to 2010. Just as the $\tau = 5$ calculations shown in Fig. 2, the difference between the return eigenvalue and the market-adjusted return eigenvalue is significant. In this case, the market-adjusted return eigenvalue is *strictly* less than the return eigenvalue. This indicates that the market-adjusted return modification reduces the influence of the "market" and its corresponding eigenvector even on long timescales. Though the analogs of Figs. 3 and 4 are not shown for $\tau = 100$, their implications also hold for this experiment. Like Fig. 3, Fig. 6 shows the corresponding empirical probability densities for the eigenvalues from both methods.

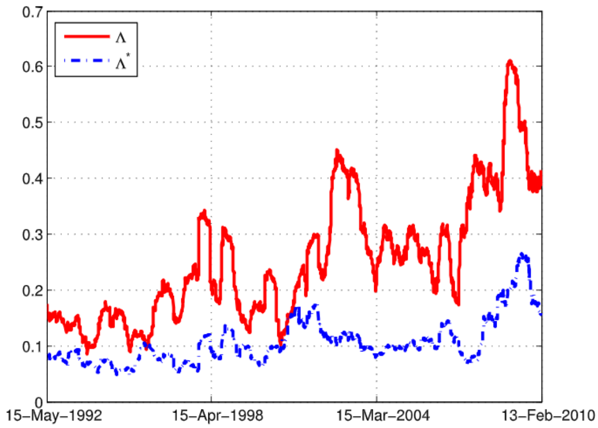

Fig. 5. Time series comparison of Λ and Λ^{*} for window size $\tau = 100$, 1991–2010.

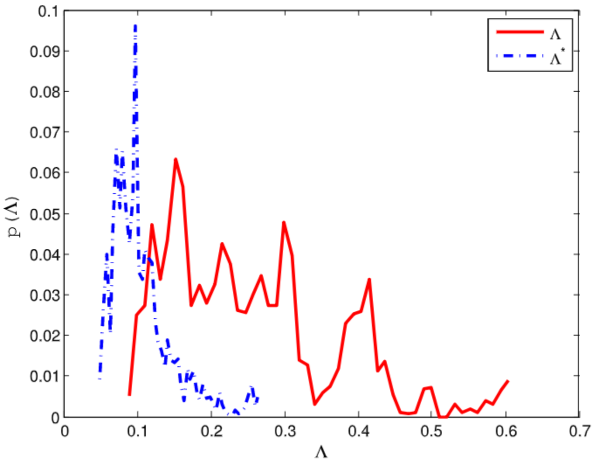

Fig. 6. Distribution of Λ and Λ^{*} for window size $\tau = 100$, 1991–2010.

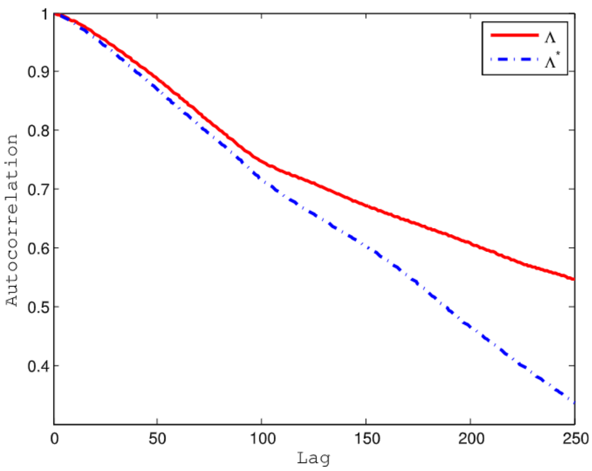

Fig. 7. Autocorrelation of Λ and Λ^{*} for window size $\tau = 100$.

Fig. 7 shows the autocorrelation of Λ and Λ^{*}. The autocorrelation function of the market-adjusted return eigenvalue is strictly less than the autocorrelation function of the return eigenvalue. This indicates that the market-adjusted eigenvalue time series is more stationary than the unadjusted return eigenvalue time series.

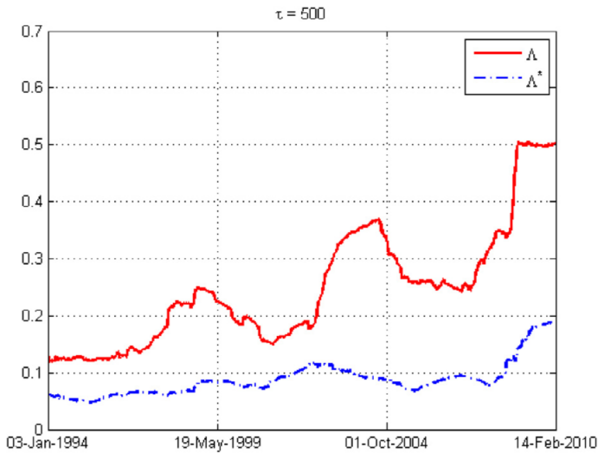

Fig. 8. Time series comparison of Λ and Λ^* for window size $\tau = 500$, 1991–2010.

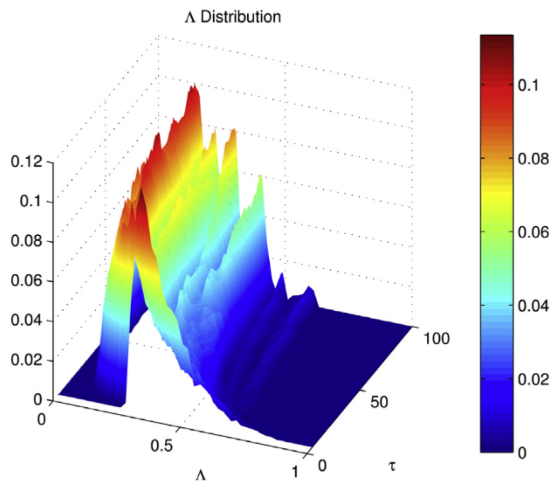

Fig. 9. 3-D distribution of Λ for $\tau \in \{5, \ldots, 100\}$.

We obtain a smoother version of Fig. 5 for both Λ and Λ^* in Fig. 8 by using large window size $\tau = 500$.

4.3. *Robustness with respect to* τ

Many results may be sensitive to certain parameter values. In order to ensure that our results above are not purely a function of our choices of τ, we analyze the sensitivity of the eigenvalue distributions to values of $\tau \in \{5, \ldots, 100\}$. This analysis covers the region between the distributions shown above in Figs. 3 and 6, thereby creating a surface of stitched-together distributions as τ varies.

Figs. 9 and 10 show the surface representing the distribution of Λ for various values of τ. Fig. 9 has six combined or single peak layers. The highest peaked layer has a smooth L-shape with turning near origin, while the others are relatively more parallel. In Fig. 10, each horizontal slice of the surface represents a fixed value of τ, sums to 1, and corresponds in meaning to Figs. 3 and 6 above. This figure clearly demonstrates the multi-modality of the return eigenvalue series across all values of τ. Interestingly, a number of elevated regions run vertically along the figure and the curvature of the region seems to be inversely related to the region's mean value of Λ.

Fig. 11 shows the surface representing the three dimensional distribution of Λ^* for various values of τ. Each horizontal slice of the surface again represents a fixed value of τ, sums to 1, and corresponds in meaning to Figs. 3 and 6. Fig. 11 shows a much tighter distribution than Fig. 9. Though elevated regions do exist for above-average values of Λ^*, these regions have much less curvature than those in Fig. 9. Furthermore, the color scale in Fig. 11 clearly shows that the peak values of Λ^* on the surface approach 0.30, whereas the corresponding peak of the scale in Fig. 9 only approaches 0.15.

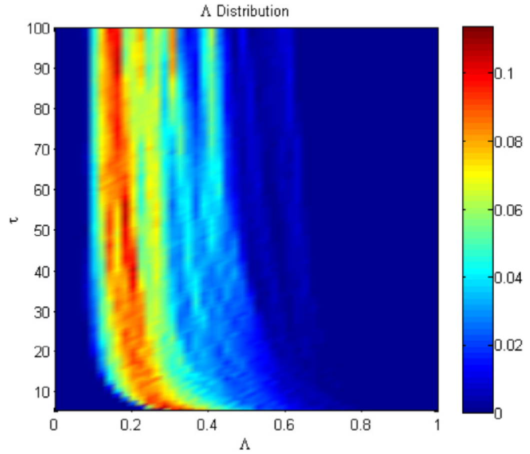

Fig. 10. Distribution of Λ for $\tau \in \{5, \dots, 100\}$.

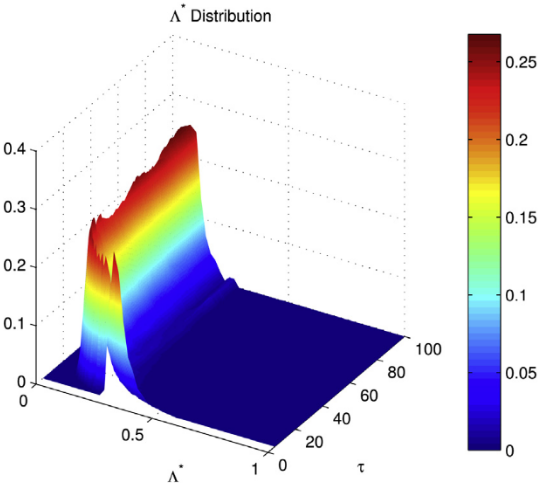

Fig. 11. 3-D distribution of Λ^* for $\tau \in \{5, \dots, 100\}$. (For interpretation of the references to color in this figure legend, the reader is referred to the web version of this article.)

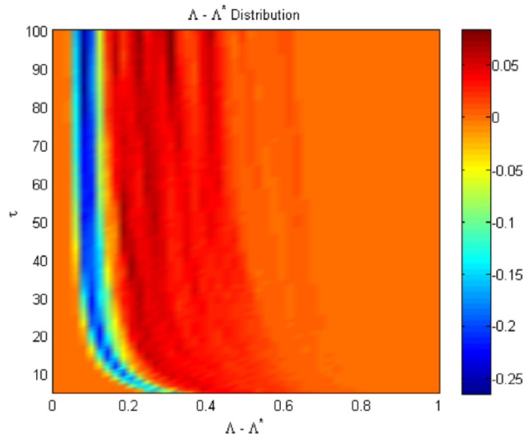

Fig. 12. The difference of distributions for $\tau \in \{5, \dots, 100\}$.

To compare these surfaces in Figs. 9 and 11, Fig. 12 shows the difference surface corresponding to the distributions of Λ and Λ^*. We find that the distribution of Λ has larger values than the distribution of Λ^* on the plain domain of normalized maximum eigenvalue versus τ, except for the smooth L-shaped strip.

5. Conclusion

In this paper, we present a method for calculating the eigenvalues of a time-dependent correlation matrix based on market-adjusted return. First, we compare our method to a previous method using return eigenvalue on an 18-year dataset of 310 S&P 500-listed stocks for two fixed correlation window sizes. We find that the resulting eigenvalue time series is relatively more stationary in mean value and standard deviation than the unadjusted time series. While the unadjusted time series standard deviations are 0.1080 and 0.1146 for $\tau = 5$ and $\tau = 100$ respectively, the corresponding adjusted time series standard deviations are 0.0442 and 0.0424. Moreover, the market-adjusted return eigenvalue shows more peakedness in that the kurtosis of the market-adjusted return eigenvalue distribution is 6.22, nearly twice the kurtosis value of 3.79 for the return distribution. Furthermore, the market-adjusted return eigenvalue's autocorrelation function value (ACF) is strictly less than the ACF of the return eigenvalue. The market-adjusted return ACF decreases faster than the return ACF at τ as the lag increases.

Later, to ensure that our results are not dependent on the choice of these window sizes, we perform this analysis for each window size $\tau \in \{5, \ldots, 100\}$. This again supports that the distribution of the eigenvalue time series for our market-adjusted return is more stationary than that produced by the previous method. When we compare the three dimensional distributions of Λ and Λ^*, we find that the distribution of Λ has larger values than the distribution of Λ^* except for the smooth L-shaped strip domain. The L-shaped domain is an approximate polarization domain. The market-adjusted return eigenvalue is less than the return eigenvalue for most of the time period. One financial interpretation can be that there is co-movement of individual stock returns and market around 84% of time among the maximum proportion of variance explained by market. The remaining time may corresponds to relatively more polarization.

It is valuable to examine polarization domains where comovement of financial variables may turn out to be far from each other. For example, [33] and [34] found and analyzed a kind of polarization of stock price return and interest rates in US stock markets and BIST-100 index for some sufficiently large time intervals, unlike the Heston model [35] suggests. Krugman, 2008 nobel laureate in economics, suggested low interest rates close to zero in order to encourage investments (see Krugman [36]), that the Heston model cannot capture.

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